MATH 651 : GENERAL TOPOLOGY
Fall 2014

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Lectures MWF 9:30 in Wean Hall 7218
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Text: Lecture notes will be provided as we go along. Supplementary text: *Topology* (2nd edition) by James Munkres 2000, Prentice Hall.

Learning Objectives. In short, you will learn the mathematical tools to deal with general spaces, understand their features, as well as those of mappings between spaces. This will be done in significant generality and depth. As these notions are essential to analysis, applied mathematics, differential geometry, and algebraic topology, this course will prepare you for studies of these subjects. In particular, many notions (such as convergence, continuity, compactness, connectedness) studied in depth in this course are part of the language in which statement of analysis and related subjects are made. You will learn about two fundamental ways is which closeness is described: via a distance (metric), or, more generally, using neighborhoods (topology). You will also learn important properties of the notions introduced that will enable you to apply them in the areas mentioned above.

Prerequisites. Knowledge of real analysis (at the level of an undergraduate course such as Math 355). Familiarity with notions of metric spaces and uniform convergence is recommended.

Evaluation. The course grade will be based on problem sets (50%), a midterm exam (20%), and a final exam (30%). Midterm exam will take place on Monday, October 13th. The exams are closed book exams, however everyone will be allowed to bring one (two sided) letter sized sheet of notes.

Problem sets. There will be 7 problem sets. The problem sets and due dates will be posted on the course blackboard page. Late homework will not receive score. However, if you have a valid reason for not doing a problem set (illness for example), the particular homework will not count towards your grade.
Discussing the problem sets with your classmates is fine, as long as you are only exchanging ideas and general knowledge, and not the solutions to the problems. In particular everyone should present his/her own solutions.

APPROXIMATE OUTLINE

- Finite and infinite sets
- Ordering, Zorn's lemma
- **Topological Spaces** (definition and examples)
- Interior and closure
- Base and subbase of a topology; neighborhood base
- Countability axioms
- Topological constructions: Induced (i.e. relative) topology, Direct image, Inverse image, Quotient topology, Product topology (finite)
- Metric spaces (definition and examples)
- Limit point
- Sequences
- Separability
- Functions and continuity
- Connectedness, pathwise connectedness, connected components
- Separation axioms (Hausdorff, regular and normal spaces)
- Compactness, Compactification, Alexandroff compactification
- Product topology, Tychonoff theorem
- Stone–Čech compactification.
- Limit, continuity revisited
- Limsup, liminf, lower semicontinuity
- Normal spaces, Urysohn lemma, Tietze extension theorem.
- Paracompact spaces and partitions of unity, Michael theorem.
- Metrization, Urysohn metrization theorem,
- Nagata–Smirnov metrization theorem (time permitting)
- **Topology of Metric Spaces**
- Cauchy sequences, completeness, Baire category theorem, completion of a metric space
- uniform continuity
- Banach contraction principle
- Compactness, sequential compactness, total boundedness, finite intersection property Ascoli–Arzelá theorem
- Dini's theorem, Stone–Weierstrass theorem
- Brouwer's Fixed Point Theorem
- Fundamental group (time permitting)