MATH 499 UNDERGRADUATE RESEARCH TOPIC

MATHEMATICS OF IMAGE PROCESSING

FALL 2015

Course webpage: Will be available on CMU Blackboard
Lectures MWF 1:30 in WEH 5312
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Prerquisites. Math 241 (or 242) , 261 (or 260), 268 (or 269, or 259), 355 (or math studies). Math 369 and 356, and 372 are welcome, but not required.

Description. The course focuses on PDE and variational approaches to image processing and applications to data analysis. First we will consider classical problems of image processing: denoising, deblurring, segmentation, and inpainting. We will introduce the mathematical tools and background needed to model these problems (i.e. describe them as a mathematical problem). We will then investigate how to solve the mathematical problems obtained. This includes an introduction to parabolic partial differential equations (like heat equation), and calculus of variations (minimizing relevant functionals). Furthermore numerical approaches will be discussed. We will not be able to give a fully rigorous introduction into these subjects. The idea is to develop some conceptual understanding and appreciation for what the problems are, what can be done and what you may want to study further.
We will then turn to ways to compare (i.e. quantify similarity) between images. We will study deformation based approaches. This will lead us to study basic notions of Riemannian geometry and optimal transportation. We remark that many of the problems do not have a universal solution, but that what is the best depends on the setting and particular question asked. A number of problems with opportunity for independent study and research will be presented.

Learning objectives. The course will introduce you to a very active field of applied mathematics which has many connections and uses. You will learn about mathematical techniques needed to denoise, deblur, segment, inpaint and compare images. This includes learning about partial differential equations, minimization (calculus of variations), Riemannian geometry, optimal transportation and numerical approaches to solve the problems. You will learn how to take an applied problem and pose it in mathematical terms. You will be able to analyze if the mathematical description is a viable one and suggest approaches to solve it. You will also gain an experience on doing mathematical research. Namely how to approach a situation when there is much that you do not know, including parts of background material. This is in contrast to a typical homework problems which are usually closely connected to a

narrow subject covered in the lectures. This is particularly the case with the final project which will prepare you for dealing with larger and open-ended tasks.
The course will also enable you to write simple code in MATLAB and run numerical experiments using MATLAB.

Evaluation. The course grade will be based on problem sets (50%) and final project (50%). The homework sets will include both mathematical problems and programing tasks (for example to numerically implement an algorithm). However the emphasis is on understanding what algorithms do rather than on efficient numerical implementation.

Problem sets will be posted regularly on the course's Blackboard webpage. There will always be at least one week between the problem posting and the solution due date. Late homework will not receive score. However, if you have a valid reason for not doing a problem set (illness for example), the particular homework will not count towards your grade. Discussing the problem sets with your classmates is fine, as long as you are only exchanging ideas and general knowledge, and not the solutions to the problems. In particular everyone must present his/her own solutions.