1 Problems

2012/A1. Let \(d_1, d_2, \ldots, d_{12}\) be real numbers in the open interval \((1, 12)\). Show that there exist distinct indices \(i, j, k\) such that \(d_i, d_j, d_k\) are the side lengths of an acute triangle.

2012/A2. Let \(*\) be a commutative and associative binary operation on a set \(S\). Assume that for every \(x\) and \(y\) in \(S\), there exists \(z\) in \(S\) such that \(x * z = y\). (This \(z\) may depend on \(x\) and \(y\).) Show that if \(a, b, c\) are in \(S\) and \(a * c = b * c\), then \(a = b\).

2012/A3. Let \(f : [-1, 1] \to \mathbb{R}\) be a continuous function such that

(i) \(f(x) = \frac{2-x^2}{2} f \left( \frac{x^2}{2-x^2} \right)\) for every \(x\) in \([-1, 1]\),

(ii) \(f(0) = 1\), and

(iii) \(\lim_{x \to 1^-} \frac{f(x)}{\sqrt{1-x}}\) exists and is finite.

Prove that \(f\) is unique, and express \(f(x)\) in closed form.