1 Problems

2012/A1. Let $d_1, d_2, \ldots, d_{12}$ be real numbers in the open interval $(1, 12)$. Show that there exist distinct indices $i, j, k$ such that $d_i, d_j, d_k$ are the side lengths of an acute triangle.

2012/A2. Let $\ast$ be a commutative and associative binary operation on a set $S$. Assume that for every $x$ and $y$ in $S$, there exists $z$ in $S$ such that $x \ast z = y$. (This $z$ may depend on $x$ and $y$.) Show that if $a, b, c$ are in $S$ and $a \ast c = b \ast c$, then $a = b$.

2012/A3. Let $f : [-1, 1] \rightarrow \mathbb{R}$ be a continuous function such that

\begin{enumerate}
  \item $f(x) = \frac{2-x^2}{x^2} f \left( \frac{x^2}{2-x^2} \right)$ for every $x$ in $[-1, 1]$,
  \item $f(0) = 1$, and
  \item $\lim_{x \to 1^-} \frac{f(x)}{\sqrt{1-x}}$ exists and is finite.
\end{enumerate}

Prove that $f$ is unique, and express $f(x)$ in closed form.