1 Problems

**Putnam 2009/B4.** Say that a polynomial with real coefficients in two variables, $x, y$, is balanced if the average value of the polynomial on each circle centered at the origin is 0. The balanced polynomials of degree at most 2009 form a vector space $V$ over $\mathbb{R}$. Find the dimension of $V$.

**Putnam 2009/B5.** Let $f : (1, \infty) \to \mathbb{R}$ be a differentiable function such that
\[
f'(x) = \frac{x^2 - f(x)^2}{x^2(f(x)^2 + 1)} \quad \text{for all } x > 1.
\]
Prove that $\lim_{x \to \infty} f(x) = \infty$.

**Putnam 2009/B6.** Prove that for every positive integer $n$, there is a sequence of integers $a_0, a_1, \ldots, a_{2009}$ with $a_0 = 0$ and $a_{2009} = n$ such that each term after $a_0$ is either an earlier term plus $2^k$ for some nonnegative integer $k$, or of the form $b \mod c$ for some earlier positive terms $b$ and $c$. [Here $b \mod c$ denotes the remainder when $b$ is divided by $c$, so $0 \leq (b \mod c) < c$.]