1 Problems

**Putnam 2007/B4.** Let $n$ be a positive integer. Find the number of pairs $P, Q$ of polynomials with real coefficients such that

$$(P(X))^2 + (Q(X))^2 = X^{2n} + 1$$

and $\deg P > \deg Q$.

**Putnam 2007/B5.** Let $k$ be a positive integer. Prove that there exist polynomials $P_0(n), P_1(n), \ldots, P_{k-1}(n)$ (which may depend on $k$) such that for any integer $n$,

$$\left\lceil \frac{n}{k} \right\rceil^k = P_0(n) + P_1(n) \left\lceil \frac{n}{k} \right\rceil + \cdots + P_{k-1}(n) \left\lceil \frac{n}{k} \right\rceil^{k-1}.$$

($\lfloor a \rfloor$ means the largest integer $\leq a$.)

**Putnam 2007/B6.** For each positive integer $n$, let $f(n)$ be the number of ways to make $n!$ cents using an unordered collection of coins, each worth $k!$ cents for some $k$, $1 \leq k \leq n$. Prove that for some constant $C$, independent of $n$,

$$n^{n^2/2 - Cn} e^{-n^2/4} \leq f(n) \leq n^{n^2/2 + Cn} e^{-n^2/4}.$$