1 Problems

Putnam 2005/A1. Show that every positive integer is a sum of one or more numbers of the form $2^r3^s$, where $r$ and $s$ are nonnegative integers and no summand divides another. (For example, $23 = 9 + 8 + 6$.)

Putnam 2005/A2. Let $S = \{(a, b) | a = 1, 2, \ldots, n, b = 1, 2, 3\}$. A rook tour of $S$ is a polygonal path made up of line segments connecting points $p_1, p_2, \ldots, p_{3n}$ in sequence such that

(i) $p_i \in S$,
(ii) $p_i$ and $p_{i+1}$ are a unit distance apart, for $1 \leq i < 3n$,
(iii) for each $p \in S$ there is a unique $i$ such that $p_i = p$.

How many rook tours are there that begin at $(1, 1)$ and end at $(n, 1)$?

Putnam 2005/A3. Let $p(z)$ be a polynomial of degree $n$, all of whose zeros have absolute value 1 in the complex plane. Put $g(z) = p(z)/z^n$. Show that all zeros of $g'(z) = 0$ have absolute value 1.