1 Problems

Putnam 2004/A4. Show that for any positive integer $n$, there is an integer $N$ such that the product $x_1x_2\cdots x_n$ can be expressed identically in the form

$$x_1x_2\cdots x_n = \sum_{i=1}^{N} c_i(a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n)^n$$

where the $c_i$ are rational numbers and each $a_{ij}$ is one of the numbers $-1, 0, 1$.

Putnam 2004/A5. An $m \times n$ checkerboard is colored randomly: each square is independently assigned red or black with probability $1/2$. We say that two squares, $p$ and $q$, are in the same connected monochromatic component if there is a sequence of squares, all of the same color, starting at $p$ and ending at $q$, in which successive squares in the sequence share a common side. Show that the expected number of connected monochromatic regions is greater than $mn/8$.

Putnam 2004/A6. Suppose that $f(x, y)$ is a continuous real-valued function on the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$. Show that

$$\int_0^1 \left( \int_0^1 f(x, y) \, dx \right)^2 \, dy + \int_0^1 \left( \int_0^1 f(x, y) \, dy \right)^2 \, dx$$

$$\leq \left( \int_0^1 \int_0^1 f(x, y) \, dx \, dy \right)^2 + \int_0^1 \int_0^1 (f(x, y))^2 \, dx \, dy.$$