1 Problems

**Putnam 2007/B1.** Let $f$ be a nonconstant polynomial with positive integer coefficients. Prove that if $n$ is a positive integer, then $f(n)$ divides $f(f(n) + 1)$ if and only if $n = 1$.

**Putnam 2007/B2.** Suppose that $f : [0, 1] \to \mathbb{R}$ has a continuous derivative and that $\int_0^1 f(x) \, dx = 0$. Prove that for every $\alpha \in (0, 1)$,
\[
\left| \int_0^\alpha f(x) \, dx \right| \leq \frac{1}{8} \max_{0 \leq x \leq 1} |f'(x)|.
\]

**Putnam 2007/B3.** Let $x_0 = 1$ and for $n \geq 0$, let $x_{n+1} = 3x_n + \lfloor x_n \sqrt{5} \rfloor$. In particular, $x_1 = 5$, $x_2 = 26$, $x_3 = 136$, $x_4 = 712$. Find a closed-form expression for $x_{2007}$. ([$a$] means the largest integer $\leq a$.)