1 Problems

Putnam 2006/B1. Show that the curve \( x^3 + 3xy + y^3 = 1 \) contains only one set of three distinct points \( A, B, \) and \( C \), which are vertices of an equilateral triangle, and find its area.

Putnam 2006/B2. Prove that for every set \( X = \{x_1, \ldots, x_n\} \) of real numbers, there exists a non-empty subset \( S \) of \( X \) and an integer \( m \) such that
\[
\left| m + \sum_{s \in S} s \right| \leq \frac{1}{n+1}.
\]

Putnam 2006/B3. Let \( S \) be a finite set of points in the plane. A linear partition of \( S \) is an unordered pair \( \{A, B\} \) of subsets of \( S \) such that \( A \cup B = S \), \( A \cap B = \emptyset \), and \( A \) and \( B \) lie on opposite sides of some straight line disjoint from \( S \) (\( A \) or \( B \) may be empty). Let \( L_S \) be the number of linear partitions of \( S \). For each positive integer \( n \), find the maximum of \( L_S \) over all sets \( S \) of \( n \) points.