

Putnam E.11

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1 Problems

Putnam 2003/A1. Let n be a fixed positive integer. How many ways are there to write n as a sum of positive integers, $n = a_1 + a_2 + \cdots + a_k$, with k an arbitrary positive integer and $a_1 \leq a_2 \leq \cdots \leq a_k \leq a_1 + 1$? For example, with $n = 4$ there are four ways: 4, 2+2, 1+1+2, 1+1+1+1.

Putnam 2003/A2. Let a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n be nonnegative real numbers. Show that

$$\begin{aligned} & (a_1 a_2 \cdots a_n)^{1/n} + (b_1 b_2 \cdots b_n)^{1/n} \\ & \leq [(a_1 + b_1)(a_2 + b_2) \cdots (a_n + b_n)]^{1/n}. \end{aligned}$$

Putnam 2003/A3. Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

for real numbers x .