1 Problems

**Putnam 1987/B4.** Let \((x_1, y_1) = (0.8, 0.6)\) and let \(x_{n+1} = x_n \cos y_n - y_n \sin y_n\) and \(y_{n+1} = x_n \sin y_n + y_n \cos y_n\) for \(n = 1, 2, 3, \ldots\). For each of \(\lim_{n \to \infty} x_n\) and \(\lim_{n \to \infty} y_n\), prove that the limit exists and find it or prove that the limit does not exist.

**Putnam 1987/B5.** Let \(O_n\) be the \(n\)-dimensional vector \((0, 0, \cdots, 0)\). Let \(M\) be a \(2n \times n\) matrix of complex numbers such that whenever \((z_1, z_2, \ldots, z_{2n})M = O_n\), with complex \(z_i\), not all zero, then at least one of the \(z_i\) is not real. Prove that for arbitrary real numbers \(r_1, r_2, \ldots, r_{2n}\), there are complex numbers \(w_1, w_2, \ldots, w_n\) such that

\[
\text{re} \begin{pmatrix}
M & \begin{pmatrix}
w_1 \\
\vdots \\
w_n
\end{pmatrix}
\end{pmatrix} = \begin{pmatrix}
r_1 \\
\vdots \\
r_{2n}
\end{pmatrix}.
\]

(Note: if \(C\) is a matrix of complex numbers, \(\text{re}(C)\) is the matrix whose entries are the real parts of the entries of \(C\).)

**Putnam 1987/B6.** Let \(F\) be the field of \(p^2\) elements, where \(p\) is an odd prime. Suppose \(S\) is a set of \((p^2 - 1)/2\) distinct nonzero elements of \(F\) with the property that for each \(a \neq 0\) in \(F\), exactly one of \(a\) and \(-a\) is in \(S\). Let \(N\) be the number of elements in the intersection \(S \cap \{2a : a \in S\}\). Prove that \(N\) is even.

**IMC 1996/B1 (Bonus random analysis problem - work on if finished early).** Let \(f : [0, 1] \to [0, 1]\) be a continuous function. Let \(x_1, x_2, \ldots\) be a sequence satisfying \(x_{n+1} = f(x_n)\) for all \(n \geq 1\), and suppose that

\[
\lim_{n \to \infty} (x_{n+1} - x_n) = 0.
\]

Prove that the entire sequence \(x_1, x_2, \ldots\) converges.