1 Problems

Putnam 1978/B2. Find
\[ \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{1}{i^2 j + 2 ij + ij^2}. \]

Putnam 1978/B3. Let the polynomials \( p_n(x) \) be defined by
\begin{align*}
p_1(x) &= 1 + x \\
p_2(x) &= 1 + 2x \\
p_{2n+1}(x) &= p_{2n}(x) + (n + 1)x \cdot p_{2n-1}(x) \\
p_{2n+2}(x) &= p_{2n+1}(x) + (n + 1)x \cdot p_{2n}(x).
\end{align*}
Let \( a_n \) be the largest real root of \( p_n(x) \). Prove that \( a_n \) is monotonic increasing and tends to zero.

Putnam 1978/B4. Show that we can find integers \( a, b, c, d \) such that
\[ a^2 + b^2 + c^2 + d^2 = abc + abd + acd + bcd, \]
and the smallest of \( a, b, c, d \) is arbitrarily large.