1 Problems

**Putnam 1978/A1.** Show that every 20-element subset of \( \{1, 4, 7, 10, 13, 16, \ldots, 100\} \) contains two distinct elements which sum to 104.

**Putnam 1978/A2.** Let \( A \) be the real \( n \times n \) matrix with diagonal entries \( c_1, c_2, \ldots, c_n \), all entries above the diagonal equal to \( a \), and all entries below the diagonal equal to \( b \), which is not equal to \( a \). Prove that

\[
\det A = \frac{bp(a) - ap(b)}{b - a},
\]

where \( p(x) = \prod_{i=1}^{n} (c_i - x) \).

**Putnam 1978/A3.** Let \( p(x) = 2(x^6 + 1) + 4(x^5 + x) + 3(x^4 + x^2) + 5x^3 \). Which of these is the smallest?

\[
\int_{0}^{\infty} \frac{x}{p(x)} \, dx \quad \int_{0}^{\infty} \frac{x^2}{p(x)} \, dx \quad \int_{0}^{\infty} \frac{x^3}{p(x)} \, dx \quad \int_{0}^{\infty} \frac{x^4}{p(x)} \, dx
\]