1 Problems

**Putnam 1994/A4.** Let $A$ and $B$ be $2 \times 2$ matrices with integer entries such that $A, A + B, A + 2B, A + 3B,$ and $A + 4B$ are all invertible matrices whose inverses have integer entries. Show that $A + 5B$ is invertible and that its inverse has integer entries.

**Putnam 1994/A5.** Let $(r_n)_{n \geq 0}$ be a sequence of positive real numbers such that $\lim_{n \to \infty} r_n = 0$. Let $S$ be the set of numbers representable as a sum

$$r_{i_1} + r_{i_2} + \cdots + r_{i_{1994}},$$

with $i_1 < i_2 < \cdots < i_{1994}$. Show that every nonempty interval $(a, b)$ contains a nonempty subinterval $(c, d)$ that does not intersect $S$.

**Putnam 1994/A6.** Let $f_1, \ldots, f_{10}$ be bijections of the set of integers such that for each integer $n$, there is some composition $f_{i_1} \circ f_{i_2} \circ \cdots \circ f_{i_m}$ of these functions (allowing repetitions) which maps 0 to $n$. Consider the set of 1024 functions

$$\mathcal{F} = \{f_{i_1}^{e_1} \circ f_{i_2}^{e_2} \circ \cdots \circ f_{i_{10}}^{e_{10}}\},$$

$e_i = 0$ or 1 for $1 \leq i \leq 10$. ($f_i^0$ is the identity function and $f_i^1 = f_i$.) Show that if $A$ is any nonempty finite set of integers, then at most 512 of the functions in $\mathcal{F}$ map $A$ to itself.