1 Problems

Putnam 1982/B1. \(ABC\) is an arbitrary triangle, and \(M\) is the midpoint of \(BC\). How many pieces are needed to dissect \(AMB\) into triangles which can be reassembled to give \(AMC\)?

Putnam 1982/B2. Let \(a(r)\) be the number of lattice points inside the circle centered at the origin with radius \(r\). Let \(k = 1 + e^{-1} + e^{-4} + \cdots + e^{-n^2} + \cdots\). Express

\[
\int_U a(\sqrt{x^2 + y^2})e^{-(x^2+y^2)}dxdy
\]

as a polynomial in \(k\), where \(U\) represents the entire plane.

Putnam 1982/B3. Let \(p_n\) be the probability that two numbers selected independently and randomly from \(\{1, 2, 3, \ldots, n\}\) have a sum which is a square. Find \(\lim_{n \to \infty} p_n \sqrt{n}\).