1 Riddle

Why do mathematicians always confuse Halloween and Christmas?

2 Problems

**Putnam 1983/B1.** Let $C$ be a cube side 4, center $O$. Let $S$ be the sphere center $O$ radius 2. Let $A$ be one of the vertices of the cube. Let $R$ be the set of points in $C$ but not $S$, which are closer to $A$ than to any other vertex of $C$. Find the volume of $R$.

**Putnam 1983/B2.** Let $f(n)$ be the number of ways of representing $n$ as a sum of powers of 2 with no power being used more than 3 times. For example, $f(7) = 4$ (the representations are $4 + 2 + 1$, $4 + 1 + 1 + 1$, $2 + 2 + 2 + 1$, $2 + 2 + 1 + 1 + 1$). Can we find a real polynomial $p(x)$ such that $f(n) = \lfloor p(n) \rfloor$?

**Putnam 1983/B3.** Let $y_1$, $y_2$, and $y_3$ be solutions of $y''' + a(x)y'' + b(x)y' + c(x)y = 0$ such that $y_1^2 + y_2^2 + y_3^2 = 1$ for all $x$. Find constants $\alpha$ and $\beta$ such that $y_1'(x)^2 + y_2'(x)^2 + y_3'(x)^2$ is a solution of $y' + \alpha a(x)y + \beta c(x) = 0$. 