Putnam E.5

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1 Problems

Putnam 1985/A1. Determine, with proof, the number of ordered triples (A_1, A_2, A_3) of sets which have the property that

(i) $A_1 \cup A_2 \cup A_3 = \{1, 2, \ldots, 10\}$, and

(ii) $A_1 \cap A_2 \cap A_3 = \emptyset$.

Express the answer in the form $2^a 3^b 5^c 7^d$, where a, b, c, and d are nonnegative integers.

Putnam 1985/A2. Let T be an acute triangle. Inscribe a rectangle R in T such that the bottom edge of R is on the base of T, and the two top corners of R touch the sides of T. Inscribe another rectangle S by placing the bottom edge of S on the top edge of R, and the top corners of S on the sides of T. Let $A(X)$ denote the area of polygon X. Find the maximum value, or show that no maximum exists, of $\frac{A(R)+A(S)}{A(T)}$, where T ranges over all triangles and R, S over all rectangles.

Putnam 1985/A3. Let d be a real number. For each integer $m \geq 0$, define a sequence $\{a_m(j)\}$, $j = 0, 1, 2, \ldots$ by the condition

$$a_m(0) = d/2^m, \quad \text{and} \quad a_m(j+1) = (a_m(j))^2 + 2a_m(j), \quad j \geq 0.$$

Evaluate $\lim_{n\to\infty} a_n(n)$.