1 Problems

Putnam 1985/A1. Determine, with proof, the number of ordered triples \((A_1, A_2, A_3)\) of sets which have the property that

(i) \(A_1 \cup A_2 \cup A_3 = \{1, 2, \ldots, 10\}\), and

(ii) \(A_1 \cap A_2 \cap A_3 = \emptyset\).

Express the answer in the form \(2^a 3^b 5^c 7^d\), where \(a\), \(b\), \(c\), and \(d\) are nonnegative integers.

Putnam 1985/A2. Let \(T\) be an acute triangle. Inscribe a rectangle \(R\) in \(T\) such that the bottom edge of \(R\) is on the base of \(T\), and the two top corners of \(R\) touch the sides of \(T\). Inscribe another rectangle \(S\) by placing the bottom edge of \(S\) on the top edge of \(R\), and the top corners of \(S\) on the sides of \(T\). Let \(A(X)\) denote the area of polygon \(X\). Find the maximum value, or show that no maximum exists, of \(\frac{A(R)+A(S)}{A(T)}\), where \(T\) ranges over all triangles and \(R, S\) over all rectangles.

Putnam 1985/A3. Let \(d\) be a real number. For each integer \(m \geq 0\), define a sequence \(\{a_m(j)\}\), \(j = 0, 1, 2, \ldots\) by the condition

\[
a_m(0) = \frac{d}{2^m}, \quad \text{and} \quad a_m(j+1) = (a_m(j))^2 + 2a_m(j), \quad j \geq 0.
\]

Evaluate \(\lim_{n \to \infty} a_n(n)\).