1 Problems

Putnam 1986/B1. Inscribe a rectangle of base $b$ and height $h$ in a circle of radius one. Further inscribe an isosceles triangle of base $b$ between the $b$-side of the rectangle and the minor arc of the circle that it determines. For what value of $h$ do the rectangle and triangle have the same area?

Putnam 1986/B2. Prove that there are only a finite number of possibilities for the ordered triple $T = (x - y, y - z, z - x)$, where $x$, $y$, and $z$ are complex numbers satisfying the simultaneous equations

$$x(x - 1) + 2yz = y(y - 1) + 2zx = z(z - 1) + 2xy,$$

and list all such triples $T$.

Putnam 1986/B3. Let $\Gamma$ consist of all polynomials in $x$ with integer coefficients. For $f$ and $g$ in $\Gamma$ and $m$ a positive integer, let $f \equiv g \pmod{m}$ mean that every coefficient of $f - g$ is an integral multiple of $m$. Let $n$ and $p$ be positive integers with $p$ prime. Given that $f$, $g$, $h$, $r$, and $s$ are in $\Gamma$ with $rf + sg \equiv 1 \pmod{p}$ and $fg \equiv h \pmod{p}$, prove that there exist $F$ and $G$ in $\Gamma$ with $F \equiv f \pmod{p}$, $G \equiv g \pmod{p}$, and $FG \equiv h \pmod{p^n}$.