1 Problems

Putnam 1987/B1. Evaluate
\[ \int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x) + \sqrt{\ln(x+3)}}} \, dx. \]

Putnam 1987/B2. Let \( r, s, \) and \( t \) be integers with \( 0 \leq r, 0 \leq s, \) and \( r+s \leq t. \) Prove that
\[ \binom{s}{0} \binom{t}{r} + \binom{s}{1} \binom{t}{r+1} + \cdots + \binom{s}{s} \binom{t}{r+s} = \frac{t+1}{(t+1-s)(t-r)}. \]

Putnam 1987/B3. Let \( F \) be a field in which \( 1+1 \neq 0. \) Show that the set of solutions to the equation \( x^2 + y^2 = 1 \) with \( x \) and \( y \) in \( F \) is given by \((x, y) = (1, 0)\) and
\[ (x, y) = \left( \frac{r^2 - 1}{r^2 + 1}, \frac{2r}{r^2 + 1} \right), \]
where \( r \) runs through the elements of \( F \) such that \( r^2 \neq -1. \)