1. Warm-up

Putnam 2012/A0. When and where is the Putnam?

1913 entrance exam to Carnegie Institute of Technology (Math). A spherical triangle has angles of 70°, 90°, and 100°, and the underlying sphere has radius 10. What is the area of the spherical triangle?

1913 entrance exam to CIT (English). What is the feminine form of the noun “duck”?

2. Problems

Putnam 1941/A2. Define \( f(x) = \int_0^x \sum_{i=0}^{n-1} \frac{(x-t)^i}{i!} dt \). Calculate the \( n \)-th derivative \( f^{(n)}(x) \).

Putnam 1942/A3. Does \( \sum_{n \geq 0} \frac{n!}{(n+1)^n} \) converge or diverge for \( k = \frac{19}{7} \)?

Putnam 1941/B3. Let \( y_1 \) and \( y_2 \) be any two linearly independent solutions to the differential equation \( y'' + p(x)y' + q(x)y = 0 \). Let \( z = y_1y_2 \). Find the differential equation satisfied by \( z \).

Putnam 1955/B2. Let \( f : \mathbb{R} \to \mathbb{R} \) be a twice differentiable function, with \( f'' \) continuous and \( f(0) = 0 \). Define \( g : \mathbb{R} \to \mathbb{R} \) by \( g(x) = f(x)/x \) for \( x \neq 0 \), and \( g(0) = f'(0) \). Show that \( g \) is differentiable and that \( g' \) is continuous.

Putnam 1949/A6. Show that \( \prod_{n=1}^{\infty} \frac{1+2\cos(2z/3^n)}{3} = \frac{\sin z}{z} \) for all complex \( z \).

Putnam 1948/B6. Take the origin \( O \) of the complex plane to be the vertex of a cube, so that \( OA, OB, OC \) are edges of the cube (with \( A, B, C \) possibly lying in the third dimension, outside the complex plane). Let the feet of the perpendiculars from \( A, B, C \) to the complex plane be the complex numbers \( u, v, w \). Show that \( u^2 + v^2 + w^2 = 0 \).

Putnam 1948/A5. Let \( \omega_1, \omega_2, \ldots, \omega_n \) be the \( n \)-th roots of unity. Find \( \prod_{i<j} (\omega_i - \omega_j)^2 \).

Putnam 1940/B6. The \( n \times n \) matrix \( (m_{ij}) \) is defined as \( m_{ij} = a_i a_j \) for \( i \neq j \), and \( a_i^2 + k \) for \( i = j \). Show that \( \det(m_{ij}) \) is divisible by \( k^{n-1} \) and find its other factor.

3. No homework

Please do not submit write-ups for any problems. There is no homework for next week. There is no next week. Do not pass Go, do not collect $200.