1 Problems

Putnam 2000/B4. Let \( f(x) \) be a continuous function such that \( f(2x^2 - 1) = 2xf(x) \) for all \( x \). Show that \( f(x) = 0 \) for \(-1 \leq x \leq 1\).

Putnam 2000/B5. Let \( S_0 \) be a finite set of positive integers. We define finite sets \( S_1, S_2, \ldots \) of positive integers as follows: the integer \( a \) is in \( S_{n+1} \) if and only if exactly one of \( a - 1 \) or \( a \) is in \( S_n \). Show that there exist infinitely many integers \( N \) for which \( S_N = S_0 \cup \{ N + a : a \in S_0 \} \).

Putnam 2000/B6. Let \( B \) be a set of more than \( 2^{n+1}/n \) distinct points with coordinates of the form \((\pm 1, \pm 1, \ldots, \pm 1)\) in \( n \)-dimensional space with \( n \geq 3 \). Show that there are three distinct points in \( B \) which are the vertices of an equilateral triangle.