1 Problems

**Putnam 2002/B4.** An integer \( n \), unknown to you, has been randomly chosen in the interval \([1, 2002]\) with uniform probability. Your objective is to select \( n \) in an odd number of guesses. After each incorrect guess, you are informed whether \( n \) is higher or lower, and you must guess an integer on your next turn among the numbers that are still feasibly correct. Show that you have a strategy so that the chance of winning is greater than \( \frac{2}{3} \).

**Putnam 2002/B5.** A palindrome in base \( b \) is a positive integer whose base-\( b \) digits read the same backwards and forwards; for example, 2002 is a 4-digit palindrome in base 10. Note that 200 is not a palindrome in base 10, but it is the 3-digit palindrome 242 in base 9, and 404 in base 7. Prove that there is an integer which is a 3-digit palindrome in base \( b \) for at least 2002 different values of \( b \).

**Putnam 2002/B6.** Let \( p \) be a prime number. Prove that the determinant of the matrix

\[
\begin{pmatrix}
x & y & z \\
x^p & y^p & z^p \\
x^{p^2} & y^{p^2} & z^{p^2}
\end{pmatrix}
\]

is congruent modulo \( p \) to a product of polynomials of the form \( ax + by + cz \), where \( a, b, c \) are integers. (We say two integer polynomials are congruent modulo \( p \) if corresponding coefficients are congruent modulo \( p \).)