1 Problems

Putnam 1988/B1. A composite (positive integer) is a product $ab$ with $a$ and $b$ not necessarily distinct integers in $\{2, 3, 4, \ldots\}$. Show that every composite is expressible as $xy + xz + yz + 1$, with $x, y, z$ positive integers.

Putnam 1988/B2. Prove or disprove: If $x$ and $y$ are real numbers with $y \geq 0$ and $y(y + 1) \leq (x + 1)^2$, then $y(y − 1) \leq x^2$.

Putnam 1988/B3. For every $n$ in the set $\mathbb{N} = \{1, 2, \ldots\}$ of positive integers, let $r_n$ be the minimum value of $|c − d\sqrt{3}|$ for all nonnegative integers $c$ and $d$ with $c + d = n$. Find, with proof, the smallest positive real number $g$ with $r_n \leq g$ for all $n \in \mathbb{N}$.

Putnam 1988/B4. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive real numbers, then so is $\sum_{n=1}^{\infty} (a_n)^{n/(n+1)}$. 