Putnam C.14

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1 Problems

Putnam 1988/B1. A *composite* (positive integer) is a product ab with a and b not necessarily distinct integers in $\{2, 3, 4, \dots\}$. Show that every composite is expressible as $xy + xz + yz + 1$, with x, y, z positive integers.

Putnam 1988/B2. Prove or disprove: If x and y are real numbers with $y \geq 0$ and $y(y+1) \leq (x+1)^2$, then $y(y-1) \leq x^2$.

Putnam 1988/B3. For every n in the set $\mathbb{N} = \{1, 2, \dots\}$ of positive integers, let r_n be the minimum value of $|c - d\sqrt{3}|$ for all nonnegative integers c and d with $c + d = n$. Find, with proof, the smallest positive real number g with $r_n \leq g$ for all $n \in \mathbb{N}$.

Putnam 1988/B4. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive real numbers, then so is $\sum_{n=1}^{\infty} (a_n)^{n/(n+1)}$.