1 Problems

**Putnam 1990/B1.** Find all real-valued continuously differentiable functions $f$ on the real line such that for all $x$,

$$(f(x))^2 = \int_0^x [(f(t))^2 + (f'(t))^2] \, dt + 1990.$$ 

**Putnam 1990/B2.** Prove that for $|x| < 1$, $|z| > 1$,

$$1 + \sum_{j=1}^{\infty} (1 + x^j)P_j = 0,$$

where $P_j$ is

$$\frac{(1 - z)(1 - zx)(1 - zx^2) \cdots (1 - zx^{j-1})}{(z - x)(z - x^2)(z - x^3) \cdots (z - x^j)}.$$

**Putnam 1990/B3.** Let $S$ be a set of $2 \times 2$ integer matrices whose entries $a_{ij}$ (1) are all squares of integers and, (2) satisfy $a_{ij} \leq 200$. Show that if $S$ has more than 50387 ($= 15^4 - 15^2 - 15 + 2$) elements, then it has two elements that commute.