General strategy

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1 Problems

Putnam 2010/A0. When and where is the Putnam?

Putnam 1992/B1. Let S be a set of n distinct real numbers. Let A_S be the set of numbers that occur as averages of two distinct elements of S. For a given $n \geq 2$, what is the smallest possible number of elements in A_S?

Plünnecke's Inequality. Let $S + S$ denote the set of all numbers which can be achieved as the sum of two (not necessarily distinct) elements of S. Suppose that $|S + S| \leq k|S|$. Let nS denote the set of all numbers which can be achieved as the sum of n (not necessarily distinct elements of S. Then for any integer n, it is always true that $|nS| \leq k^n|S|$.

Putnam 1994/A1. Suppose that a sequence $a_1, a_2, a_3, \ldots$ satisfies $0 < a_n \leq a_{2n} + a_{2n+1}$ for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

Putnam 1996/A1. Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without the interiors of the squares overlapping). You may assume that the sides of the squares will be parallel to the sides of the rectangle.

Putnam 1999/A2. Let $p(x)$ be a polynomial that is nonnegative for all real x. Prove that for some k, there are polynomials $f_1(x), \ldots, f_k(x)$ such that

$$p(x) = \sum_{j=1}^{k} (f_j(x))^2.$$

Putnam 1993/B2. Consider the following game played with a deck of $2n$ cards numbered from 1 to $2n$. The deck is randomly shuffled and n cards are dealt to each of two players, A and B. Beginning with A, the players take turns discarding one of their remaining cards and announcing its number. The game ends as soon as the sum of the numbers on the discarded cards is divisible by $2n + 1$. The last person to discard wins the game. Assuming optimal strategy by both A and B, what is the probability that A wins?