# Games

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## 1 Problems

**Putnam 2008/A2.** Alan and Barbara play a game in which they take turns filling entries of an initially empty $2008 \times 2008$ array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?

**VTRMC 2004/4.** A $9 \times 9$ chess board has two squares from opposite corners and its central square removed (so 3 squares on the same diagonal are removed, leaving 78 squares). Is it possible to cover the remaining squares using dominoes, where each domino covers two adjacent squares? Justify your answer.

**Putnam 2008/A3.** Start with a finite sequence $a_1, a_2, \ldots, a_n$ of positive integers. If possible, choose two indices $j < k$ such that $a_j$ does not divide $a_k$, and replace $a_j$ and $a_k$ by their GCD and LCM, respectively. Prove that if this process is repeated, it must eventually stop and the final sequence does not depend on the choices made.

**Classical.** Prove that the second player does not have a winning strategy for $3 \times 3$ tic-tac-toe. How about for $4 \times 4 \times 4$ tic-tac-toe?

**Putnam 2002/B2.** Consider a polyhedron with at least five faces such that exactly three edges emerge from each of its vertices. Two players play the following game:

Each player, in turn, signs his or her name on a previously unsigned face. The winner is the player who first succeeds in signing three faces that share a common vertex.

Show that the player who signs first will always win by playing as well as possible.

## 2 Bonus problem

**Putnam 2002/B4.** An integer $n$, unknown to you, has been randomly chosen in the interval $[1, 2002]$ with uniform probability. Your objective is to select $n$ in an **odd** number of guesses. After each incorrect guess, you are informed whether $n$ is higher or lower, and you **must** guess an integer on your next turn among the numbers that are still feasibly correct. Show that you have a strategy so that the chance of winning is greater than $2/3$.

**Putnam 2002/A4.** In Determinant Tic-Tac-Toe, Player 1 enters a 1 in an empty $3 \times 3$ matrix. Player 0 counters with a 0 in a vacant position, and play continues in turn until the $3 \times 3$ matrix is completed with five 1's and four 0's. Player 0 wins if the determinant is 0 and player 1 wins otherwise. Assuming both players pursue optimal strategies, who will win and how?