Induced subgraphs with distinct size or order

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For a graph G, let $\phi(G)$ denote the number of distinct pairs $(|V(H)|, |E(H)|)$, as H ranges over all induced subgraphs of G. Let $hom(G)$ denote the maximum number of vertices in an induced clique or an induced independent set in G. An n-vertex graph is *c-Ramsey*, if $hom(G) \leq c \log n$. Erdős, Faudree and Sós conjectured that *for every positive constant c, there is a positive constant $b = b(c)$ such that if G is a c-Ramsey graph on n vertices, then* $\phi(G) \geq bn^{5/2}$. They also proved the weaker lower bound $\Omega(n^{3/2})$.

We prove the bound $\Omega(n^2)$ for the wider class of n-vertex graphs with average degree at least $0.001n$ and at most $0.999n$. The order of magnitude of the bound cannot be improved in this class as shown, for example, by the complete bipartite n-vertex graphs.

This is joint work with N. Alon.