On a random graph evolving by degrees.

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Abstract. We consider a random (multi)graph growth process $\{G_m\}$ on a vertex set $[n]$, which is a special case of a more general process suggested by Laci Lovász several years ago. G_0 is empty, and G_{m+1} is obtained from G_m by inserting a new edge e at random. Specifically, the conditional probability that e joins two currently disjoint vertices, i and j, is proportional to $(d_i+\alpha)(d_j+\alpha)$, where d_i, d_j are the degrees of i, j in G_m, and $\alpha>0$ is a fixed parameter. The limiting case $\alpha=\infty$ is the Erdős-Rényi graph process. We show that whp G_m contains a unique giant component iff $c:=2m/n>c_\alpha=\alpha/(1+\alpha)$, and the size of this giant is asymptotic to $n\left[1-\left(\frac{\alpha+c^*}{\alpha+c}\right)^\alpha\right]$, where $c^*<c_\alpha$ is the root of $\frac{c}{(\alpha+c)^{2+\alpha}}=\frac{c^*}{(\alpha+c^*)^{2+\alpha}}$. For the multigraph version, $\{MG_m\}$, we show that MG_m is connected whp iff $m\gg m_n:=n^{1+\alpha^{-1}}$, and we conjecture that, for $\alpha>1$, m_n is the threshold for connectedness of G_m itself.