Discrete Mathematics 21-228: Summer II 2000
Homework 5
Due Friday, 7/28

July 26, 2000

You may discuss the problems in groups of at most 3 people, but each person must write her own homework. Please list the members of your group. You may not use any sources other than the notes and texts listed on the course page.

Assume graphs are simple unless otherwise noted.

1. Suppose that G is a connected simple graph that is not a complete graph. Prove that every vertex of G belongs to some 3-vertex induced subgraph isomorphic to P_3.

2. A connected graph is called unicyclic if it has exactly one cycle. Prove that G is unicyclic if and only if it satisfies any two of the following properties.

   (i) G is connected.

   (ii) G has exactly one cycle.

   (iii) The number of vertices in G equals the number of edges.

Extra Credit: Show that if G is connected and each degree in G is even, then for every $v \in V$, $G - v$ has at most $d(v)/2$ components.