Discrete Mathematics 21-228: Summer II 2000
Homework 3
Due Friday, 7/21

July 17, 2000

You may discuss the problems in groups of at most 3 people, but each person must write her own homework. Please list the members of your group. You may not use any sources other than the notes and texts listed on the course page.

Don't forget the initial conditions when finding a recurrence.

1. **Recurrence mania**
   Solve the following recurrences by any means you see fit. If you don't use the recurrence techniques we developed in class, make sure you prove your solution is correct.

   (i) The recurrence for the number of regions in the plane that n lines in general position (every 2 meet at a point, but no 3 do) define:
   $$p_n = p_{n-1} + n,\ n \geq 1;\ p_0 = 1.$$

   (ii) The recurrence for the number of regions in space that n planes in general position (every 3 meet at a point, but no 4 do) define:
   $$s_n = s_{n-1} + \frac{n^2 - n + 2}{2},\ n \geq 1;\ s_0 = 1.$$

2. **Permutations**
   Let e_n be the number of permutations of $[n] = \{1, 2, \ldots, n\}$ which do not contain odd cycles. Show that
   $$e_n = (n-1)^2 e_{n-2},\ n \geq 2.$$

3. **Inclusion-Exclusion**
   Count the number of 5-digit extensions, where each digit may be 0-9 unless otherwise specified, which satisfy the following properties.

   (i) The sum of the digits is 9.

   (ii) The sum of the digits is 10.

   (iii) The sum of the digits is 30.

   (iv) The sum of the digits is 30, and the first digit is either a 2 or an 8.

Extra Credit: Son of inclusion-exclusion

Each person attending a party is to bring a prize. The host has set up a series of contests for distribution of the prizes. The people at this party are generally greedy so anyone can win any number of prizes.

(i) In how many ways can the prizes be distributed?

(ii) In how many ways can the prizes be distributed if no person leaves with the prize (s)he brought?