Discrete Mathematics 21-228: Summer II 2000
Homework 1
Due Monday, 7/10

July 7, 2000

You may discuss the problems in groups of at most 3 people, but each person must write her own homework. Please list the members of your group. You may not use any sources other than the notes and texts listed on the course page.

1. In class we explored that problem of distributing k identical balls in n distinct bins (bins are numbered $1, 2, \ldots, n$) and found there were $\binom{k+n-1}{k}$ ways to accomplish this. What if we have restrictions on the number of balls in each bin? Find the number of solutions if we want:

   a) at least 1 ball in each bin

   b) at least i balls in bin i

   c) an even number of balls in each bin

   d) an odd number of balls in each bin

2. How many binary strings of length n have exactly k 1's such that no two are consecutive? For $n = 2$ we have 00, 01, and 10 but not 11. (Hint: Can our pals Ball and Bin from above help?)

3. Show that

$$1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \cdots + n^2\binom{n}{n}^2 = n^2\binom{2n-2}{n-1}.$$

   You may use any of the formulas from the notes. If you decide to approach this combinatorially, here's a hint: Relate the expression to the number of ways of choosin 2 equal sized sets $A, B \subset \{1, 2, \ldots, n\}$ and choosing one "special" element from each of A and B.