# 6.1.3. Primal or Dual?

a) \( \text{Max } Z = 10x_1 - 4x_2 + 7x_3 \)
\[ \begin{align*}
\text{s.t.} & \quad 3x_1 - x_2 + 2x_3 \leq 25 \\
& \quad x_1 - 2x_2 + 3x_3 \leq 25 \\
& \quad 5x_1 + x_2 + 2x_3 \leq 40 \\
& \quad x_1 + x_2 + x_3 \leq 90 \\
& \quad 2x_1 - x_2 + x_3 \leq 20 \\
& \quad x_1, x_2, x_3 \geq 0 
\end{align*} \]

It is easier to solve the question with fewer constraints. This primal problem has 5 constraints, but dual question only needs to deal with 3 constraints. Therefore, it is simpler to apply simplex method on the dual problem.

b) It is easier to solve the question with fewer constraints. Primal problem has 2 constraints, while dual problem has 5 constraints. Therefore, it is simpler to apply simplex method on the primal problem.

# 6.1.7. \( \text{Max } Z = x_1 + 2x_2 \)
\[ \begin{align*}
\text{s.t.} & \quad -x_1 + x_2 \leq -2 \quad \Rightarrow \quad x_1 - x_2 \geq 2 \\
& \quad 4x_1 + x_2 \leq 4 \quad \Rightarrow \quad 4x_1 + x_2 \leq 4 \\
& \quad x_1, x_2 \geq 0 
\end{align*} \]

There's no common feasible region, then this problem doesn't have feasible soln.
b) Primal
\[ \begin{align*}
\text{max } Z &= x_1 + 2x_2 \\
\text{S.t. } &\begin{align*}
-x_1 + x_2 &\leq -2 \\
4x_1 + x_2 &\leq 4 \\
x_1, x_2 &\geq 0
\end{align*}
\end{align*} \]

Dual:
\[ \begin{align*}
\text{min } W &= -2y_1 + 4y_2 \\
&\begin{align*}
-y_1 + 4y_2 &\geq 1 \\
y_1 + y_2 &\geq 2 \\
y_1, y_2 &\geq 0
\end{align*}
\end{align*} \]

The feasible region (shaded) is unbounded, which graphically showed that the dual problem has an unbounded objective function. We can be infinitely small, since \( y_1 \) can be infinitely large.

6.1.11 Use weak duality property to prove that if both the primal and dual problems have feasible soln, then they must both have an optimal soln.

Primal
\[ \begin{align*}
\text{max } Z &= x_1 \\
\text{S.t. } &\begin{align*}
x_1 \leq b \\
x_1 \geq 0
\end{align*}
\end{align*} \]

Dual
\[ \begin{align*}
\text{min } W &= y^T b \\
\text{S.t. } &\begin{align*}
y^T A &\geq c \\
y &\geq 0
\end{align*}
\end{align*} \]

By Weak Duality Theorem, \( CX^* \leq y^* b \), which implies that \( cx \leq y^* b \), and \( CX^* \leq y^* b \).

\[ \Rightarrow \] primal solns are bounded below the dual solns, and the dual solns are bounded above primal solns

\[ \Rightarrow \] feasible region for primal problem is bounded below dual solns, and the feasible region for dual problem is bounded above primal solns.

\[ \Rightarrow \] Their objective functions are not unbounded.

\[ \Rightarrow \] Since both of their feasible region and objective functions are bounded, then there must exist optimal solutions for both problems.
# 44. \[ \text{max} \ 2x_1 + x_2 \]

**s.t.:**

\[ x_1 + x_2 \leq 5 \]
\[ 3x_1 + x_2 \leq 10 \Rightarrow \]
\[ x_1, x_2 \geq 0. \]

**max** \[ 2x_1 + x_2 \]

**s.t.:**

\[ x_1 + x_2 + x_3 = 5 \]
\[ 3x_1 + x_2 + x_4 = 10. \]
\[ x_1, x_2, x_3, x_4 \geq 0. \]

\[ \text{a) Dual} \]

\[ \text{mm} \ 5y_1 + 10y_2 \]
\[ y_1 + 3y_2 \geq 2 \]
\[ y_1 + y_2 \geq 1 \]
\[ y_1, y_2 \geq 0. \]

\[ \begin{align*}
&y_2 \\vdots \\
&y_2 = \frac{2}{3} - \frac{5}{3} y_1 \\vdots \\
&0 \leq y_1, y_2 \leq \infty
\end{align*} \]

\[ \begin{align*}
&x_1, x_2, x_3, x_4 \\vdots \\
&x_1, x_2, x_3, x_4 \geq 0 \\
&\text{1st} \ (0, 0, 5, 10), Z = 0. \\
&\text{2nd} \ (\frac{10}{3}, 0, 5, 0), Z = \frac{20}{3} \\
&\text{3rd} \ (\frac{5}{2}, \frac{5}{2}, 0, 0), Z = \frac{15}{2}
\end{align*} \]

\[ \begin{align*}
&y_1 \\vdots \\
&y_1 = \frac{2}{3} - \frac{1}{2} y_2 \\vdots \\
&0 \leq y_1, y_2 \leq \infty
\end{align*} \]

\[ \begin{align*}
&x_1, x_2 \\vdots \\
&x_1, x_2 \geq 0 \\
&\text{1st} \ (-2, -1, 0, 0), W = 0 = Z \\
&\text{2nd} \ (0, \frac{1}{3}, 0, \frac{1}{3}), W = \frac{20}{3} = Z \\
&\text{3rd} \ (0, 0, \frac{1}{2}, \frac{1}{2}), W = \frac{15}{2} = Z
\end{align*} \]

\[ \begin{align*}
&x_1, x_2, x_3, x_4 \\vdots \\
&x_1, x_2, x_3, x_4 \geq 0
\end{align*} \]

\[ \begin{align*}
&\text{1st} \ -2 - 1 - 0 = \frac{1}{3} \\
&\text{2nd} \ 1 - 5 - 3 = \text{RHS}
\end{align*} \]

\[ \begin{align*}
&\text{3} - 1 - 10 \\
&\frac{1}{3} \text{ RHS}
\end{align*} \]

\[ \begin{array}{cccccc}
& 0 & -\frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\
& 0 & 2 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
& 1 & \frac{1}{3} & - \frac{1}{3} & 1 & \frac{1}{3} \text{ RHS}
\end{array} \]

\[ \begin{align*}
&0, 0, 0, 0 \\
&\frac{15}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}
\end{align*} \]