21-292. HW5

10/10

6.1.3. Primal or Dual?

a) Max $Z = 10X_1 - 4X_2 + 7X_3$

s.t. $3X_1 - X_2 + 2X_3 \le 25$

$X_1 - 2X_2 + 3X_3 \le 25$

$5X_1 + X_2 + 2X_3 \le 40$

$X_1 + X_2 + X_3 \le 90$

$2X_1 - X_2 + X_3 \le 20$

$X_1, X_2, X_3 \geq 0$.

It is easier to solve the question with fewer constraints. This primal problem has 5 constraints, but dual question only needs to deal with 3 constraints. Therefore, it is simpler to apply simplex method on the dual problem.

2/2

b) It is easier to solve the question with fewer constraints, Primal problem has 2 constraints, while dual problem has 5 constraints. Therefore, it is simpler to apply simplex method on the primal problem.

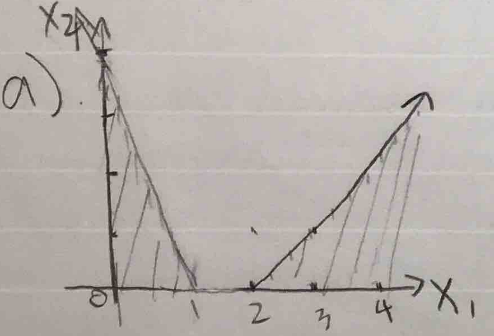
6.1.7. max $Z = X_1 + 2X_2$

s.t. $-X_1 + X_2 \le -2 \Rightarrow X_1 - X_2 \geq 2$

$4X_1 + X_2 \le 4 \qquad 4X_1 + X_2 \le 4$

$X_1, X_2 \geq 0 \qquad X_1, X_2 \geq 0$.

a)



There're no common feasible region. Then this problem doesn't have feasible soln.

$X_2 \le 4 - 4X_1$

b) Primal

max $Z = x_1 + 2x_2$

s.t. $-x_1 + x_2 \le -2$

$4x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$.

$\Rightarrow$

Dual.

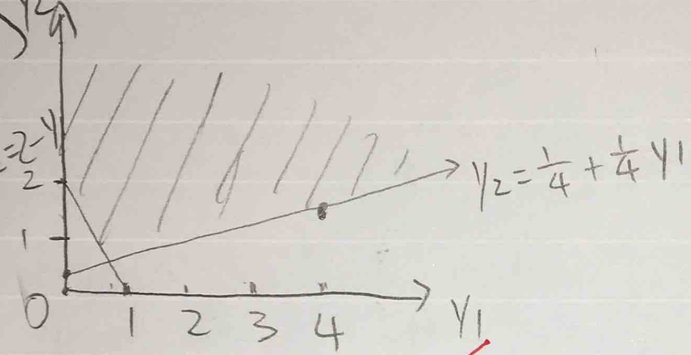
min $W = -2y_1 + 4y_2$

$-y_1 + 4y_2 \ge 1$

$y_1 + y_2 \ge 2$

$y_1, y_2 \ge 0$.

c)



2/2

The feasible region (shaded) is unbounded, which graphically showed that the dual problem has an unbounded objective function.

W can be infinitely small, since y_1 can be infinitely large.

#6.1.11 Use weak duality property to prove that if both the primal and dual problem have feasible soln, then they must both have an optimal soln.

1/1

Primal

max $\vec{c}\vec{x}$

s.t $A\vec{x} \le \vec{b}$

$\vec{x} \ge 0$

Dual

min $\vec{y}\vec{b}$

S.t. $\vec{y}A \ge \vec{c}$

$\vec{y} \ge 0$.

By Weak Duality Theorem, $cx^* \le y^*b$,

which implies that $cx \le y^*b$, and $cx^* \le yb$.

$\Rightarrow$ primal solns are bounded below the dual solns, and the dual solns are bounded above primal solns

$\Rightarrow$ feasible region for primal problem is bouned below dual solns, and the feasible region for dual problem is bounded above primal solns.

$\Rightarrow$ Their objective functions are not unbounded.

$\Rightarrow$ Since both of their feasible region and objective functions are bounded, then there must exist optimal solutions for both problems.

#4. $\max\ 2x_1 + x_2$ s.t. $x_1 + x_2 \le 5$, $3x_1 + x_2 \le 10$, $x_1, x_2 \ge 0$ $\Rightarrow$ $\max\ 2x_1 + x_2$, $x_1 + x_2 + x_3 = 5$, $3x_1 + x_2 + x_4 = 10$, $x_1, x_2, x_3, x_4 \ge 0$.

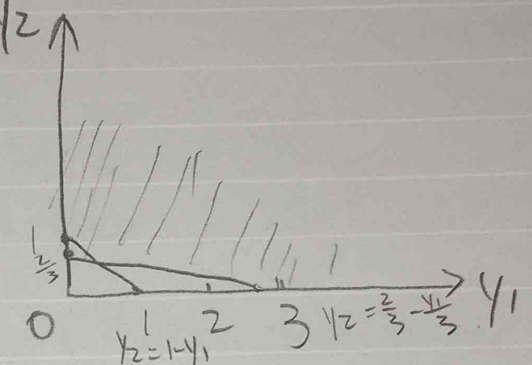
a) Dual

$\min\ 5y_1 + 10y_2$

$y_1 + 3y_2 \ge 2$

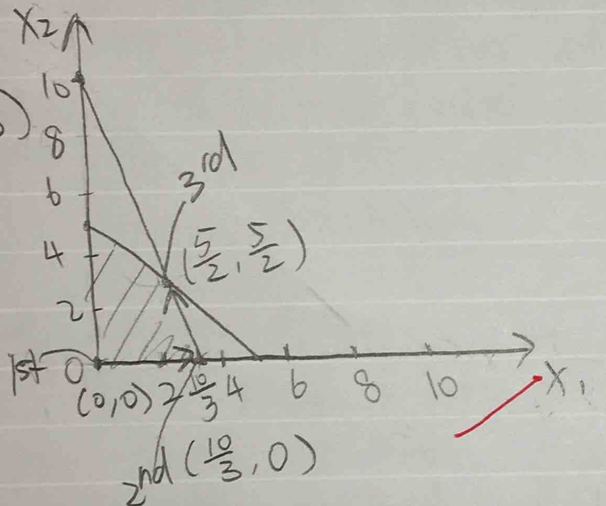
$y_1 + y_2 \ge 1$

$y_1, y_2 \ge 0$



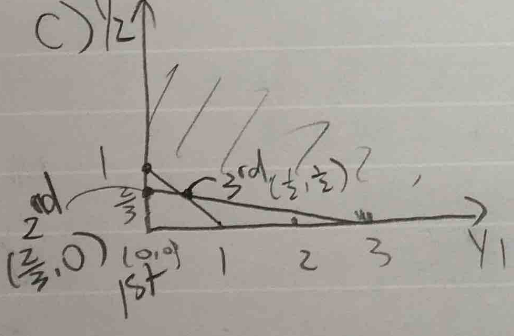
5/5

b)



	x_1	x_2	x_3	x_4	
1st	(0,	0,	5,	10),	$z = 0$
2nd	($\frac{10}{3}$,	0,	5,	0),	$z = \frac{20}{3}$
3rd	($\frac{5}{2}$,	$\frac{5}{2}$,	0,	0),	$z = \frac{15}{2}$

c)



1st $(-2, -1, 0, 0)$, $w = 0 = z$

2nd $(0, -\frac{1}{3}, 0, \frac{2}{3})$, $w = \frac{20}{3} = z$

3rd $(0, 0, \frac{1}{2}, \frac{1}{2})$ $w = \frac{15}{2} = z$

1st
$$\begin{array}{ccc} x_1 & x_2 & \\ -2 & -1 & 0 = z \\ 1 & 1 & 5 \\ 3 & 1 & 10 \end{array} \Big\} \text{RHS}$$

$\Downarrow$

2nd
$$\begin{array}{ccc} 0 & -1/3 & 20/3 = z \\ 0 & 2/3 & 5/3 \\ 1 & 1/3 & 10/3 \end{array} \Big\} \text{RHS}$$
$\Rightarrow$ 3rd
$$\begin{array}{ccc} 0 & 0 & 15/2 = z \\ 0 & 1 & 5/2 \\ 1 & 0 & 5/2 \end{array}$$

d) $5 + 5\varepsilon/2$

e) $2 + \varepsilon/2$