Name: ______________________ **PID:** ______________

Please circle your section:

A01 5:00p - 5:50p, TA: Thomas McCann

A02 6:00p - 6:50p, TA: Thomas McCann

A03 8:00p - 8:50a, TA: Susan Elle

A04 9:00p - 9:50a, TA: Susan Elle

A05 10:00p - 10:50a, TA: Susan Elle

A06 11:00p - 11:50a, TA: Susan Elle

MATH 10B: MIDTERM EXAM 2

Nov. 16, 2012

Do not turn the page until instructed to begin.

Turn off and put away your cell phone.

No calculators or any other devices are allowed.
You may use one 8.5×11 page of handwritten notes, but no other assistance.
Read each question carefully, answer each question completely, & show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.
Good luck!

#	Points	Score
1	16	
2	5	
3	8	
4	16	
Σ	45	

1. (a) (8 points) Evaluate the following definite integral:

$$\int_0^{\pi/4} x \sin(x)\, dx =$$

Integration by part : $u = x \qquad du = 1 \cdot dx$

$dv = \sin(x)\, dx \qquad v = -\cos x$

$$\int_0^{\pi/4} x \sin(x) dx = \int_0^{\pi/4} u\, dv = uv \Big|_0^{\pi/4} - \int_0^{\pi/4} v\, du$$

$$= -x \cos x \Big|_0^{\pi/4} + \int_0^{\pi/4} \cos x\, dx$$

$$= -\frac{\pi}{4} \cos\frac{\pi}{4} - 0 + \sin x \Big|_0^{\pi/4}$$

$$= -\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \sin(\pi/4) - 0$$

$$= -\frac{\sqrt{2}}{8}\pi + \frac{\sqrt{2}}{2}$$

(b) (8 points) Evaluate the following indefinite integral:

$$\int \frac{1}{x \ln(2x)}\, dx =$$

Integration by substitution : $u = \ln(2x)$, $du = \frac{1}{x} dx$

$$\int \frac{1}{x\ln(2x)} dx = \int \frac{1}{u} du = \ln|u| + C$$

$$= \ln|\ln(2x)| + C$$

2. (5 points) Calculate the following derivative:

$$\frac{d}{dt}\int_{\pi}^{t^2} e^{-\cos(x)}\,dx =$$

Let $f(t) = \int_{\pi}^{t} e^{-\cos(x)}\,dx$

$$\frac{d}{dt}\int_{\pi}^{t^2} e^{-\cos(x)}\,dx = \frac{d}{dt} f(t^2) = f'(t^2)\cdot(t^2)'$$

$$= 2t \cdot e^{-\cos(t^2)}$$

3. (8 points) Calculate the following integral, if it converges. Otherwise, show that it diverges. (*hint:* $\lim_{z\to\infty}\arctan(z) = \pi/2$.)

$$\int_0^\infty \frac{1}{x^2+9}\,dx =$$

$$\int_0^\infty \frac{1}{x^2+9}\,dx = \frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big|_0^\infty$$

$$= \lim_{x\to\infty}\frac{1}{3}\arctan\left(\frac{x}{3}\right) - 0$$

$$= \frac{1}{3}\cdot\frac{\pi}{2} = \frac{\pi}{6}$$

4. (a) (5 points) Use the inequality $\sin(x) \leq x$ and the fact that $\sin(x) \geq 0$ for $0 \leq x \leq 1$ to show the following improper integral converges:

$$\int_0^1 \frac{\sin(x)}{x}\, dx.$$

Since $0 \leq \sin(x) \leq x$ for $0 \leq x \leq 1$, we have

$$0 \leq \frac{\sin(x)}{x} \leq 1 \quad \text{for} \quad 0 \leq x \leq 1.$$

Therefore, $0 \leq \int_0^1 \frac{\sin(x)}{x}\, dx \leq 1 \cdot (1-0) = 1$

The improper integral converges

(b) (8 points) Use the comparison test to show the following integral *converges*:

$$\int_0^1 \frac{1}{(y^3+y)^{1/3}}\, dy.$$

$$\frac{1}{(y^3+y)^{1/3}} \leq \frac{1}{(y)^{1/3}} = \frac{1}{y^{1/3}} \quad \text{for} \quad 0 \leq y \leq 1$$

By comparison test, $\int_0^1 \frac{1}{(y^3+y)^{1/3}}\, dy$ ~~diverges~~ ~~converges~~ converges since

$\int_0^1 \frac{1}{y^{1/3}}\, dy$ converges

(c) (3 points) Based on the information in (b) and the fact that the integral $\int_1^\infty \frac{1}{(y^3+y)^{1/3}}\, dy$ *diverges*, determine whether the following integral converges or diverges. Explain your reasoning.

$$\int_0^\infty \frac{1}{(y^3+y)^{1/3}}\, dy.$$

Diverge. $\int_0^\infty \frac{1}{(y^3+y)^{1/3}}\, dy = \int_0^1 \frac{1}{(y^3+y)^{1/3}}\, dy + \int_1^\infty \frac{1}{(y^3+y)^{1/3}}\, dy$