Approval Voting

21-301 2018

3 May

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All of the voting methods we have yet discussed are what is known as "preferential voting" methods. In a preferential voting system, voters are asked for their specific preferences regarding candidates: who do you prefer most, who in second place, etc. Nonpreferential voting is a variant in which voters do not rank candidates, but instead divide into categories of "acceptable" and "unacceptable." The following schema include some elements of nonpreferential voting, although some combine preferential and nonpreferential ideas.

1.1 Approval Voting: Basics

In an approval voting system, each voter indicates, for each candidate, whether they approve of that choice or not. There is no ranking necessary. The candidate that then has the highest number of approvals goes on to win the election.

Approval voting has gotten some recent support in political circles as a way of preventing a so-called "spoiler" candidate from taking support away from an otherwise viable first-choice candidate. If voters are permitted to approve multiple candidates, then they can vote both for the major candidate and the minor "spoiler." In addition, proponents of approval voting say that such systems reduce negative campaigning, as your goal is not to become a "lesser of two evils" but to maximize the number of voters who approve of you (this claim is hotly debated, but we've veered now into the crossover between mathematics and behavioral science).

Criticisms of this approach center around the tendency of voters to engage in "bullet" voting, that is, voting only for their first choice preferred candidate rather than all those they approve; and the tendency of voters to approve of a candidate that they may not actually approve in order to prevent a different candidate from winning. In other words, if I hate candidate A, I may issue a ballot that approves of all candidates other than A, in order to hurt A maximally.

There are two main regimes in which we study approval voting, based on how we think about voter preferences. The first is similar to a preferential system: we presume that each voter comes equipped with a poset that details their preferences among candidates. In the second, we assume each voter comes equipped with a score assigned to each candidate.

1.2 Approval Voting: Ordinal Preferences

In this version of approval voting, we assume voters carry with them a poset that describes their preferences among the candidates. We no longer require this poset to be a linear order, but allow that there may be candidates that voters do not compare (i.e., see as essentially equal in quality). Most of the time, we assume that voters are *sincere*. A sincere voter is subject to the following restriction: If the voter votes for candidate A, then that voter also votes for every candidate in $U(A)$, the up-set of A in the poset. Logically that makes sense, as if you prefer B to A in the poset, then if you approve of A you also would approve of B.

The study of approval voting then often focuses on where voters draw their line between candidates they approve of or not. This "line" is referred to as their approval threshold, or a cutoff. In the case that every

voter is using a linear ordering, we often use the phrase "dichotomous cutoff" here: you set a line (like 3rd place) and vote for all candidates that exceed it, and do not vote for candidates that do not exceed it. If all voters have a linear ordinal preference, and they all set the same dichotomous cutoff, this is referred to as an "absolute dichotomous cutoff."

1.3   Approval Voting: Cardinal Preferences

In this version of approval voting, we assume that voters carry with them a ranking function that assigns a score to each candidate, with higher scores meaning a higher preference toward that candidate. As with ordinal preferences, we assume that voters are sincere, in that if the voter votes for candidate A, they also will vote for all candidates with score above A in their preference.

As with ordinal preferences, we therefore must determine the dichotomous cutoff for each voter. It is common, mathematically, to set this cutoff at the average score for an individual voter. This is quantitatively sensible, as if you vote for everybody above your average score, you are effectively helping only those candidates you see as above average; essentially you are looking for a positive expected return on your vote.

In the case that the dichotomous cutoff is set to the same score for all voters, we call this, as above, an "absolute dichotomous cutoff."

2   Variations

1. **Negative Voting.** In a negative voting system, each voter has 1 vote to give. They can either give that vote to a candidate, or they can use their vote to vote against a different candidate. Think of it this way: each voter has one point, but they can choose whether to put a + or − in front of their point. They can give a positive point to someone they like, or a negative point to someone they do not like. Each candidate's score is then the sum of the votes that have been issued to them, and the candidate with the highest positive score wins. If no candidate has a positive score, then nobody wins and the election is invalid.

   Notice that negative voting is subsumed by approval voting; a −1 to candidate A is the same as an approval voting ballot in which a voter approves all candidates other than A.

2. **Disapproval Voting.** Disapproval voting is basically the opposite of plurality voting. In a plurality voting system, each voter can issue one vote for their first choice candidate, and the candidate with the most votes wins. In a disapproval voting system, each voter can issue one anti-vote for their last choice candidate, and the candidate with the least anti-votes wins. This is basically the same as looking at the last row of the preference matrix, and selecting as the winner the candidate with the fewest last place votes. Hence, disapproval voting is, as plurality voting, a preferential scheme.

   Variants on disapproval voting allow voters to select as many candidates as they wish to disapprove of. This is essentially the inverse of approval voting. Indeed, if one views an approval voting scheme as dividing the candidates into "approve" and "disapprove," then approval and disapproval voting are exactly the same.

3. **Cumulative Voting, aka Range Voting, aka Utility Voting.** In a cumulative voting scheme, voters have a fixed number of points to give out, p. They can divide their p points among the candidates in any way they might like. For example, a voter could give all p points to a single candidate (this is known as "bullet voting"), or they could give a small number of points to every candidate of whom they approve. Points are then tallied, and the candidate having the most points wins the election.

4. **Majoritarian Voting.** A variant of cumulative voting, sometimes called Majoritarian Voting, is a scheme in which each voter is forced to assign points to each candidate. It can be thought of in this way: for each candidate, assign them a score from $1 - 5$, with 5 being the best possible score. Scores are then tallied and the candidate having the highest score wins. In some versions of a majoritarian system, the candidate with the highest median score wins, rather than the highest score overall.