Math 301: Homework 1

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Complete the following problems. Fully justify each response.

1. Complete problems 9, 10, 11 from Miscellany

2. In a voting matrix, a set of candidates $\{B_1, B_2, \ldots, B_k\}$ are called *clones* if every voter ranks these candidates in some order, with no other candidates between them. For example, in the following matrix, candidates C and D are clones:

5	3	2
A	B	C
C	D	D
D	C	B
B	A	A

   Notice that C and D appear in every column in adjacent positions. One might expect to see clone-like candidates in, for example, a jungle primary, in which two Republicans and two Democrats are both running for a top-two runoff. If each voter places the two Republicans in adjacent positions, and the two Democrats in adjacent positions, then each of these pairs of candidates are clones.

   A voting system is said to be *clone positive* if adding a clone of candidate A has the following outcome: There exists a clone X of A (possibly A itself) such that X is ranked above all candidates that A was ranked above originally, and possibly more. A voting method is said to be *clone negative* if adding a clone of candidate A has the opposite outcome: for every clone X of A, X may be ranked below a candidate that A was ranked above originally, and cannot improve.

   Prove that Borda Count is clone positive, and that plurality voting is clone negative.

3. In any voting matrix, a Condorcet Cycle is a sequence of candidates $A_1, A_2, \ldots, A_k$ such that A_i wins a head-to-head contest with A_{i+1} for all i, and A_k wins a head-to-head contest with A_1. Prove that a Condorcet Cycle exists if there is no Condorcet winner. Give an example to show that there can be a Condorcet Cycle even if there is a Condorcet winner.

4. Suppose we run an approval vote in which we are assured that every voter is sincere (totally unrealistic, but this is math, not sociology).
   (a) Prove that this method will satisfy the Pareto Criterion.
   (b) Prove that if every voter makes a sincere vote with an absolute dichotomous cutoff, the method also satisfies IIA.
   (c) Why is this not a violation of Arrow's Theorem?