

Math 301: Homework 7

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Complete the following problems. Fully justify each response.

1. Let G be a graph of order $n \geq 4$ and having $m \geq \frac{n}{2}\sqrt{n-1} + 1$ edges. Prove that G contains either a 3-cycle or a 4-cycle.
2. (a) Let $G = (V, E)$ be a graph with average degree D . Show that there exists a subgraph H in G with $\delta(H) \geq \frac{D}{2}$, that is, the minimum degree in H is at least $\frac{D}{2}$.
(b) Let T be a tree on k vertices. Prove that $\frac{n(k-2)}{2} \leq ex(n, T) \leq nk$ for any $n \in \mathbb{N}$ having n divisible by $k-1$.

Note: We do not know $ex(n, T)$ for an arbitrary T . It is conjectured that the lower bound above is tight, but it is not proven. It is certainly tight for some trees (paths, stars, probably others).