Math 301: Homework 10

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Complete the following problems. Fully justify each response.

1. Let G be a graph. A game is played on G as follows. No starting position is given, and the first player chooses a vertex v_1. On each turn, players choose vertices $v_1, v_2, \ldots, v_k$ such that v_i is adjacent to v_{i-1} for all i, and no vertex may be repeated (that is, the players build a path in the graph).

   Prove that the first player will win (i.e., can choose a P-position for v_0) if and only if the graph G has no perfect matching.

2. Suppose you play a 3-pile subtraction game, with the following rules.

   - In the first pile, you may remove any even number of chips, or, if the pile has only one chip, you may remove that chip.
   - In the second pile, you may remove any number of chips divisible by three (except for exactly 3), or, if the pile has 2 (mod 3) chips, you may remove them all.
   - In the third pile, you may remove any number of chips at any time.

   Take the starting position to be $(15, 28, 17)$. Determine the value of the Sprague-Grundy function at this position. Find, if it exists, an optimal move.

3. Complete problem 4 in Game Theory: Part II on page II-8.