Math 259: Midterm 3 Review
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The midterm on December 1 covers Sections 15.1-15.3, 15.5-15.9, and 16.1-16.4. Below is a list of major topics covered on the exam. For each topic, I have included some problems for reference. At the end of this document is a set of practice problems that take similar structure to the way I would present problems in an exam setting. Please contact me with any questions.

- Double Integrals
  - Have a basic understanding of how the Riemann sum can be used to approximate volume (precise technical details not necessary)
  - Fubini's Theorem (Box 10 in Section 15.1)
  - Type I/Type II regions: review pages 1042, 1043
  - Area as a double integral: Page 1047
  - For double integrals over rectangular regions: Section 15.1: 9-11, 15-34
  - For general double integrals: Section 15.2: 1-10, 17-32
- Average Value of a function
  - Section 15.1 (there are not many problems on this, but just know how the calculation works)
- Double Integrals in Polar Coordinates
  - Know how to change from polar to rectangular and back (page 1050)
  - Know box 2 in Section 15.3
  - Section 15.3: 7-34
- Surface Area
  - Know boxes 2 and 3 in Section 15.5, and how to use these formulas
  - Section 15.5: 1-12
- Triple Integrals
  - Have a basic understanding of how the triple Riemann sum can be used to approximate a triple integral (precise technical details not necessary)
  - Type I/II/III regions: pages 1071-1074
  - Volume as a triple integral: Box 12 in Section 15.6
  - Section 15.6: 3-22, 27-28
- Cylindrical Coordinates

    - Cylindrical-rectangular conversion (Boxes 1,2 in Section 15.7)
    - Cylindrical variable change for integrals: Box 4 in Section 15.7
    - Section 15.7: 11-13, 15-25, 29, 30

- Spherical Coordinates
    - Spherical-rectangular conversion (Boxes 1, 2 in Section 15.8)
    - Spherical variable change for integrals: Box 3 in Section 15.8
    - Section 15.8: 11-30, 41-43

- Change of Variables for Double and Triple Integrals
    - Two variable change: Jacobian and integral formula (Boxes 7, 9 in Section 15.9)
    - Three variable change: Jacobian and integral formula (Boxes 12, 13 in Section 15.9)
    - Section 15.9: 7-19, 23-27

- Vector fields: basic definitions
    - Section 16.1: 11-18, 21-24, 29-34

- Line Integrals
    - Definitions: Boxes 3, 7 in Section 16.2
    - Section 16.2: 1-16

- Line Integrals over Vector Fields
    - Box 13 in Section 16.2
    - Section 16.2: 19-22, 39-46

- Fundamental Theorem for Line integrals, and determining if a vector field is conservative
    - Theorems: Boxes 2, 3, 4, 5, 6 in Section 16.3
    - Section 16.3: 3-20

- Green's Theorem
    - Basic statement: Page 1136
    - Green's Theorem for area: Box 5 in Section 16.4
    - Section 16.4: 5-14, 17-19

Practice Problems

Note: These problems are here to give you a feel for the structure of exam problems. I would NOT recommend using only these problems to study. This set of problems is not representative of the length of a true exam.

1. Set up (but do not evaluate) an iterated integral that represents each of the following. Note that it is NOT sufficient to write your answer as $\iint_D f(x,y)\,dA$, but instead you should write limits for x and y (or other variables, if you change variables.)

   (a) The volume of the solid enclosed by the paraboloid $z = 2x^2 + 2y^2 - 1$ and the planes $z = 1$, $x = \pm\frac{1}{2}$.

   (b) The volume of the solid beneath the surface $z = xy$ and above the triangle having coordinates $(1,0)$, $(0,1)$, and $(2,2)$.

   (c) The average value of the function $z = (x+y)^2$ over the rectangle $[0,2] \times [0,2]$.

   (d) The area of the region outside $r = 3\cos(\theta)$ and inside $r = 1 + \cos(\theta)$

   (e) The volume of the solid between the cone $z^2 = x^2+y^2$ and the sphere $x^2+y^2+z^2 = 4$.

   (f) The area of the portion of the hyperbolic paraboloid $z = x^2 - y^2$ between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

   (g) The area of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 2y$.

   (h) The volume bounded by the cone $z^2 = x^2 + y^2$, the cylinder $x^2 + y^2 = 16$, and the planes $z = -5$ and $z = 4$.

2. Determine the volume of the solid below the surface $z = e^{-x^2-y^2}$ above the annulus centered at $(0,0)$, with inner radius 1 and outer radius 2.

3. Calculate the volume of the solid enclosed by the surface $z = 1 + x^2ye^y$ and the planes $z = 0$, $x = \pm 1$, $y = 0$, and $y = 1$.

4. Sketch the surface whose volume is represented by the following triple integral:

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{4y^2+4z^2}^{4} dxdzdy$$

5. Suppose you have a solid ball of radius 2. Find the average distance between a point in the ball and the center of the ball.

6. Consider the hemisphere $z = \sqrt{16 - x^2 - y^2}$. The hemisphere is cut by a plane that lies at an angle of $\frac{\pi}{3}$ to the xy-plane. Determine the volume of each of the two resulting solids.

7. Calculate $\iint_D (x^2+y^2)^{3/2} dA$, where D is the region in the first quadrant of the xy-plane bounded by the curves $y = 0$, $y = \sqrt{3}x$, and $x^2 + y^2 = 9$.

8. Calculate $\iint\limits_R \frac{2y-x}{x+y}\,dA$ over the rectangle R having vertices $(-1,1)$, $(1,2)$, $(1,-1)$, and $(3,0)$.

9. Evaluate $\int\limits_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$ and C is obtained by following the parabola $y = 1 + x^2$ from the point $(-1,2)$ to the point $(1,2)$.

10. Find the work done by the force field $\mathbf{F}(x,y) = (x+2)\mathbf{i} + y\mathbf{j}$ in moving an object along an arch of the cycloid defined by $\mathbf{r}(t) = (1-\cos t)\mathbf{i} + (t - \sin t)\mathbf{j}$, $0 \le t \le 2\pi$.

11. Determine if each of the following vector fields is conservative. If so, find a function f such that $\mathbf{F} = \nabla \mathbf{f}$.

    (a) $\mathbf{F} = (2x+y)\mathbf{i} + (x+3y^2)\mathbf{j}$
    (b) $\mathbf{F} = (2x+2y)\mathbf{i} + (x+y^2)\mathbf{j}$
    (c) $\mathbf{F} = (x\cos y + 2)\mathbf{i} + (-x\sin y + 2x)\mathbf{j}$
    (d) $\mathbf{F} = (2x\cos(x+y^2) - x^2\sin(x+y^2))\,\mathbf{i} - 2x^2y\sin(x+y^2)\mathbf{j}$

12. Use Green's Theorem to calculate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = (x^2+y)\mathbf{i} + (3x-y^2)\mathbf{j}$ and C is the circle $x^2+y^2=4$.

13. Suppose $\mathbf{F}$ is a conservative vector field and C is a closed curve. Prove that $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$. You can use Green's Theorem.