Math 259: Midterm 2 Review
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The midterm on October 25 covers Sections 14.2-14.8. Below is a list of major topics covered on the exam. For each topic, I have included some problems for reference. At the end of this document is a set of practice problems that take similar structure to the way I would present problems in an exam setting. Please contact me with any questions.

- Limits in more than one variable
  - Review box 1 in Section 14.2
  - Section 14.2: problems 5-22
- Continuity in more than one variable
  - Review box 4 in Section 14.2
  - Section 14.2: problems 29-38
- Definitions/interpretations of partial derivatives
  - Review pages 914-915, 918 in Section 14.3
  - Know Clairaut's Theorem
  - Section 14.3: problems 15-44, 53-58, 63-70
- Tangent Planes to a surface
  - Review box 2 in Section 14.4
  - Section 14.4: problems 1-6
- Linear approximations for a function of two variables
  - Review pages 930, 932 in Section 14.4
  - Section 14.4: problems 11-16, 22-24, 31-39
- Chain Rule
  - Review boxes 2, 3, 4 in Section 14.5
  - Section 14.5: problems 1-14, 17-26, 35, 36, 38-40, 45-48
- Implicit differentiation for functions of the form $F(x, y) = 0$.
  - Review pages 942-943 in Section 14.5
  - Section 14.5: problems 27-34
- Directional Derivatives, gradient, and maximum rate of change
  - Review box 3 and pages 950-952 in Section 14.6

- Section 14.6: problems 7-17, 28, 30, 35, 39-40 (directional derivatives, gradient)
  - Section 14.6: problems 21-26, 31-34 (maximum rate of change)
- Tangent planes to level surfaces (i.e., surfaces of the form F(x, y, z)=k)
  - Review page 954 in Section 14.6
  - Section 14.6: problems 41-46, 51-63
- Local optimization
  - Review boxes 1, 2, 3 in Section 14.7
  - Section 14.7: problems 5-22, 41-56
- Global optimization and Lagrange Multipliers
  - Review box 8 (Extreme Value Theorem) and box 9 in Section 14.7
  - Review the box entitled "Method of Langrange Multipliers" (page 972) and page 976 in Section 14.8
  - Section 14.7: problems 31-38
  - Section 14.8: problems 3-23

Practice Problems

Note: These problems are here to give you a feel for the structure of exam problems. I would NOT recommend using only these problems to study. This set of problems is not representative of the length of a true exam.

1. For each of the following, determine if the limit exists. Show that your answer is correct.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^4 - y^2}{2x^2 + 3y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{5y^4 \sin^2(x)}{x^4 + y^4}$

   (d) $\displaystyle\lim_{(x,y)\to(3,\sqrt{\pi})} \frac{\sin(xy^2) + x}{x^2 - y^2}$

   (e) $\displaystyle\lim_{(x,y)\to(0,0)} x^4 \sin\left(\frac{x+y}{y^2 - 7}\right)$

2. Calculate all the first and second partial derivatives of the following functions.

   (a) $f(x,y) = x^2y - x + y - 3$

   (b) $f(x,y) = e^{xy} + \cos(2x + y)$

   (c) $f(x,y,z) = \ln(x + y + z^2)$

   (d) $f(x_1, x_2, \ldots, x_n) = (x_1 + x_2 + \cdots + x_n)^2$

3. Find the equation of the tangent plane to the function $f(x,y) = e^{xy} + x - y$ at the point $(1, 1, e)$.

4. A cylindrical can is made out of metal, having diameter 5cm and height 7cm. Given that the thickness of the metal is .05cm, estimate the total amount of metal used in making the can.

5. Find f_s and f_t when $f(x,y) = x^2 + 3y + e^{xy}$, $x(s,t) = s^2 + t$, and $y(s,t) = st$.

6. Find the rate of change of the function f at the indicated point P in the indicated direction $\mathbf{d}$.

   (a) $f(x,y) = x^2 + 3y^2$, $P = (1,2)$, $\mathbf{d} = \langle 2, 5 \rangle$

   (b) $f(x,y) = x^2e^{-2y}$, $P = (3,0)$, $\mathbf{d} = \langle 3, 4 \rangle$.

   (c) $f(x,y,z) = \ln(x + 2y + 3z)$, $P = (1,1,1)$, $\mathbf{d} = \langle 1, 3, 2 \rangle$

7. Suppose a hurricane is approaching the city of Miami. The wind speed of the hurricane is modeled by the function

$$f(x,y) = \frac{1}{400000}x^2y^2 - \frac{9}{20000}x^2 + \frac{9}{20}x - \frac{9}{20000}y^2 + \frac{7}{20}y - \frac{1}{100}xy + \frac{535}{8},$$

where x and y represent the distance from Miami in the E-W and N-S axes.

Suppose you are 10 miles Southwest of the city, and you drive toward the city. What is the rate of change of the windspeed as you drive? What is the direction you should drive for which the windspeed is decreasing most (or increasing least)?

8. Find an equation for the tangent plane to the surface given by $2xy + 2yz + 2xz = 20$ at the point $(2, 2, 3/2)$.

9. Find the global extrema of the following functions on the indicated regions.

   (a) $f(x, y) = x^2 + y^2 - y$ on $x^2 + y^2 \leq 2$

   (b) $f(x, y) = e^{-xy}$ on $4x^2 + y^2 \leq 4$

   (c) $f(x, y) = 2x^2 + 2x + y^2 + y$ on $x^2 + y^2 \leq 4$

10. The intersection between the plane $2x + 3y - 7z = 2$ and the ellipsoid $z^2 = x^2 + y^2$ is an ellipse. Find the points on that ellipse that are closest and farthest from the origin.

11. Show, in general, that a box having maximum volume and surface area S, where S is a constant, is always a cube.