**Math 259: Midterm 1 Review**
**Mary Radcliffe**

The midterm on September 27 covers Sections 12.1-12.6, 13.1-13.4, 14.1. Below is a list of major topics covered on the exam. For each topic, I have included some problems for reference. At the end of this document is a set of practice problems that take similar structure to the way I would present problems in an exam setting. Please contact me with any questions.

- Basics of three dimensional coordinates and vectors:
  - Section 12.1: 25-45
  - Section 12.2: 1-28, 34-46
- Dot product and its properties
  - Review Boxes 2, 3, 7 in Section 12.3
  - Section 12.3: 2-20, 23-28, 33-37
- Orthogonal Projection
  - Review page 811 in Section 12.3
  - Section 12.3: 39-52
- Cross product and its properties
  - Review boxes 8, 9, 10, 11 in Section 12.4
  - Section 12.4: 1-13, 16-26, 29-36
- Equations of Lines and Planes
  - For lines: Section 12.5: 1-22
  - For planes: Section 12.5: 23-44, 51-56, 61-63
  - For lines and planes together: Section 12.5: 45-47, 57-58
- Quadric Surfaces, graphing functions of two variables, and contour diagrams
  - Review Table 1 on page 837
  - Section 12.6: 3-8, 11-38
  - Contour diagrams: Section 14.1: 36, 38, 41-52, 61-66
  - Domains of functions: Section 14.1: 13-22
  - Graphing functions: Section 14.1: 23-31, 53, 54, 61-66
- Vector functions, and their integrals/derivatives
  - Review differentiation rules in Theorem 3 in Section 13.2

  - Section 13.1: 3-14, 21-26, 42-46
  - Section 13.2: 3-20, 23-26, 35-40

- Arc Length

  - Section 13.3: 1-6, 13-16

- Curvature

  - Section 13.3: 21-25, 32, 43-45

- **TNB** frame, associated planes

  - Section 13.3: 17-20, 47-50, 57-59

- Motion/Velocity/Acceleration

  - Review pages 874-875
  - Section 13.4: 3-16, 19-30

**Practice Problems**

Note: These problems are here to give you a feel for the structure of exam problems. I would NOT recommend using only these problems to study. This set of problems is not representative of the length of a true exam.

1. Consider the triangle having vertices $P = (1, -1, -5)$, $Q = (1, 3, -1)$, and $R = (3, -1, 3)$.
   (a) Determine the lengths of the sides of the triangle.
   (b) Determine the area of the triangle.
   (c) Is the triangle right? How do you know?

2. Let $\mathbf{x} = \langle 1, 5, -2\rangle$, $\mathbf{y} = \langle -2, 1, 3\rangle$, and $\mathbf{z} = \langle 3, 1, 0\rangle$. For each of the following expressions, determine if the expression is meaningful. If so, calculate it. If not, explain why not.
   (a) $\mathbf{x} \cdot \mathbf{y} - \mathbf{y} \cdot \mathbf{x}$
   (b) $\mathbf{x} \cdot \mathbf{y} - \mathbf{y} \times \mathbf{x}$
   (c) $\mathbf{x} \cdot (\mathbf{y} \times \mathbf{z})$
   (d) $\mathbf{x} \times (\mathbf{y} \times \mathbf{x})$
   (e) $\mathbf{x} \cdot (\mathbf{y} \times \mathbf{x})$
   (f) $(\mathbf{x} \cdot \mathbf{z}) \cdot (\mathbf{x} \cdot \mathbf{y})$
   (g) $(\mathbf{x} \times \mathbf{z}) \cdot (\mathbf{x} \times \mathbf{y})$
   (h) $\mathbf{x} + \mathbf{y} \cdot \mathbf{z}$
   (i) $\mathbf{x} + \mathbf{y} \times \mathbf{z}$
   (j) $\text{proj}_{\mathbf{y}}\, \mathbf{x}$
   (k) $\text{proj}_{\mathbf{x}\times\mathbf{y}}\, \mathbf{x}$
   (l) $\text{comp}_{\mathbf{z}}\, \mathbf{y}$
   (m) $\text{comp}_{\mathbf{z}}(\mathbf{y} \cdot \mathbf{x})$
   (n) $(\text{comp}_{\mathbf{z}}\, \mathbf{y}) \cdot \mathbf{x}$

3. Three forces act on an object. Two of these forces are at an angle of $100°$ to one another, and have magnitudes of 25 N and 12 N. The third force is perpendicular to both these, and has magnitude 4 N. Determine the force needed to exactly counterbalance these three forces. You may leave expressions like $\cos(100°)$ unsimplified.

4. Determine if the following three points are coplanar: $P = (1, 1, 5)$, $Q = (-1, -2, 0)$, $R = (0, -1, 1)$. Explain your reasoning.

5. Find equations for each of the following:
   (a) The plane through the point $(1, 3, 2)$ and parallel to the plane $2x + 4y - 3z = 56$
   (b) The line through the point $(-1, 4, 8)$ and perpendicular to plane $x - y + 4z = 0$
   (c) The line of intersection of the planes $x - y + z = 2$ and $3x + 2y + z = 1$
   (d) The plane containing the points $(1, 2, 5)$, $(-1, 1, 3)$, and $(2, 0, -4)$

6. Identify this surface. What are its traces? Roughly sketch the shape of the surface.

$$y^2 = 4x^2 + \frac{z^2}{9} + 2.$$

7. Let $\mathbf{r}(t) = \langle \cos(t), 3t + 1, 2\sin(t)\rangle$.
   (a) Sketch or describe the shape of this space curve.

(b) Find the derivative of $\mathbf{r}(t)$.

(c) Find the unit tangent vector to $\mathbf{r}(t)$ at the point $(0, \frac{3\pi}{2}+1, 2)$.

(d) Find parametric equations for the tangent line to $\mathbf{r}(t)$ at the point $(0, \frac{3\pi}{2}+1, 2)$.

8. Let $\mathbf{r}(t) = \langle e^t + t, t+1, t\cos(t)\rangle$.

   (a) Find $\mathbf{r}'(t)$.

   (b) Find a **TNB** frame for $\mathbf{r}(t)$ at the point $(1, 1, 0)$.

   (c) Find the osculating plane at the point $(1, 1, 0)$. Explain the significance of this plane.

9. Let $\mathbf{r}(t) = \langle t^2, 9t, 4t^{3/2}\rangle$ for $0 \leq t \leq 10$.

   (a) Calculate the length of the curve from time 0 to time $t$.

   (b) Calculate the curvature $\kappa(t)$.

10. Suppose a particle is in motion, having position $\mathbf{r}(t) = \langle 2\cos(2t), 2\sin(2t), t\rangle$ at time $t$, for $t \geq 0$.

    (a) Describe or sketch the shape of the curve the particle moves through.

    (b) Determine the speed the particle is moving at time $t$.

    (c) Determine the acceleration of the particle. Decompose the acceleration in terms of the tangential and normal components.

    (d) Explain what the tangential and normal components of acceleration indicate. How would these components change if the curve remained the same, but the speed of motion was increased?

11. Draw a contour plot of the function $z = (y+1)/e^x$. Describe or sketch the resulting surface.