2) a) $f_x = 2xy - 1 \quad f_y = x^2 + 1$

$f_{xx} = 2y \quad f_{yy} = 0 \quad f_{xy} = 2x = f_{yx}$

---

b) $f_x = ye^{xy} - 2\sin(2x+y) \quad f_y = xe^{xy} - \sin(2x+y)$

$f_{xx} = y^2 e^{xy} - 4\cos(2x+y) \quad f_{yy} = x^2 e^{xy} - \cos(2x+y)$

$f_{xy} = e^{xy} + y^2 e^{xy} - 2\cos(2x+y)$

---

c) $f_x = \dfrac{1}{x+y+z^2} = f_y \quad f_z = \dfrac{2z}{x+y+z^2}$

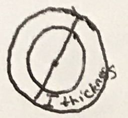
$f_{xx} = -\dfrac{1}{(x+y+z^2)^2} = f_{yy} = f_{xy} = f_{yx}$

$f_{xz} = -\dfrac{2z}{(x+y+z^2)^2} = f_{zx} = f_{yz} = f_{zy}$

$f_{zz} = \dfrac{2(x+y+z^2) - 4z^2}{(x+y+z)^2}$

---

d) $\dfrac{\partial f}{\partial x_i} = 2x_i \qquad \dfrac{\partial^2 f}{\partial x_i \partial x_j} = \begin{cases} 2 & \text{if } i=j \\ 0 & i \neq j \end{cases}$

4) 

$\Rightarrow dD = .1\text{cm}$ and $dH = .1\text{cm}$ (from top and bottom)

$D = 5\text{cm} \quad H = 7\text{cm}$

Total Amount of Metal = dSurface Area = dS

$$S = 2\pi r^2 + 2\pi r H = \pi \frac{D^2}{2} + \pi D H$$

$$\Rightarrow dS = \frac{\partial S}{\partial D} dD + \frac{\partial S}{\partial H} dH$$

$$\frac{\partial S}{\partial D} = \pi D + \pi H \qquad \frac{\partial S}{\partial H} = \pi D$$

$\Rightarrow$ Evaluate at $(5,7)$

$$\Rightarrow \frac{\partial S}{\partial D} = 12\pi \qquad \frac{\partial S}{\partial H} = 5\pi$$

$$\Rightarrow \boxed{dS = (12\pi)(.1) + 5\pi(.1) = 1.7\pi \text{ cm}^2}$$

6) a) $\nabla f = \langle 2x, 6y \rangle \Big|_{(1,2)} = \langle 2, 12 \rangle$

$d = (2,5) \Rightarrow \hat{d} = \frac{1}{\sqrt{4+25}} \langle 2, 5 \rangle$

$\Rightarrow D_d f = \frac{1}{\sqrt{29}} \left[\langle 2, 12 \rangle \cdot \langle 2, 5 \rangle \right] = \boxed{\frac{64}{\sqrt{29}}}$

---

b) $\nabla f = \langle 2x e^{-2y}, -2x^2 e^{-2y} \rangle \Big|_p = \langle 6, -18 \rangle$

$d = \langle 3, 4 \rangle \Rightarrow \hat{d} = \frac{1}{5} \langle 3, 4 \rangle$

$\Rightarrow D_d f = \frac{1}{5} \left[\langle 6, -18 \rangle \cdot \langle 3, 4 \rangle \right] = \boxed{-\frac{54}{5}}$

---

c) $\nabla f = \left\langle \frac{1}{x+2y+3z}, \frac{2}{x+2y+3z}, \frac{3}{x+2y+3z} \right\rangle \Big|_p = \frac{1}{6} \langle 1, 2, 3 \rangle$

$d = \langle 1, 3, 2 \rangle \Rightarrow \hat{d} = \frac{1}{\sqrt{1+9+4}} \langle 1, 3, 2 \rangle$

$\Rightarrow D_d f = \frac{1}{6\sqrt{14}} \left[\langle 1, 2, 3 \rangle \cdot \langle 1, 3, 2 \rangle \right] = \boxed{\frac{13}{6\sqrt{14}}}$

8) $F(x,y,z) = 2xy + 2yz + 2xz \qquad P = (2, 2, 3/2)$

$\nabla F = \langle 2y + 2z,\ 2x + 2z,\ 2y + 2x \rangle$

Evaluate at P

$\Rightarrow \nabla F = \langle 7, 7, 8 \rangle$

Tangent Plane: $\boxed{0 = 8(z - \tfrac{3}{2}) + 7(x - 2) + 7(y - 2)}$

10) We use Lagrange Multipliers to minimize distance.
& maximize

Note due to monotonicity of square root function,

it suffices to apply it to $D^2 =$ distance^2

$f(x,y,z) = x^2 + y^2 + z^2$ ← distance to origin

$g(x,y,z) = 2x + 3y - 7z = 2$

$h(x,y,z) = x^2 + y^2 - z^2 = 0$

$\nabla f = \langle 2x, 2y, 2z \rangle \quad \nabla g = \langle 2, 3, -7 \rangle$

$\nabla h = \langle 2x, 2y, -2z \rangle$

$$\begin{cases} 2x = 2\lambda + 2x\mu & ① \\ 2y = 3\lambda + 2y\mu & ② \\ 2z = -7\lambda - 2z\mu & ③ \end{cases}$$

Case 1: Assume $\lambda = 0 \Rightarrow \begin{cases} 2x = 2x\mu \\ 2y = 2y\mu \\ 2z = -2z\mu \end{cases} \Rightarrow \mu = 0 \Rightarrow x = y = z = 0$

which doesn't satisfy constraint g.

Case 2: Assume $\mu = 0$

and similarly we can show $\mu \neq 0$

Thus, $\lambda \neq 0$, $\mu \neq 0$ Then ① $\Rightarrow 2x(1-\mu) = 2\lambda \Rightarrow x = \dfrac{\lambda}{1-\mu}$

This is ok as $\mu \neq 1$ (check!)

Then ② $\Rightarrow 2y(1-\mu) = 3\lambda \Rightarrow y = \dfrac{3}{2}x$

Now Plug into Constraint g

to find:

$$2x + 3\left(\frac{3}{2}x\right) - 7z = 2$$

$$\Rightarrow \left[\frac{13}{2}x - 2\right]\frac{1}{7} = z$$

Now plug into Constraint h

$$\Rightarrow x^2 + \left(\frac{3}{2}x\right)^2 - \left(\frac{1}{7}\left(\frac{13}{2}x - 2\right)\right)^2 = 0$$

$$\Rightarrow \frac{13}{4}x^2 - \frac{1}{49}\left[\frac{169}{4}x^2 + 4 - 26x\right] = 0$$

$$\Leftrightarrow \frac{468}{196}x^2 + \frac{26}{49}x - \frac{4}{49} = 0$$

$$\Leftrightarrow x = \pm\left(\frac{7\sqrt{13} - 13}{117}\right)$$

$$\Rightarrow y = \pm\frac{3}{2}\left[\frac{7\sqrt{13} - 13}{117}\right]$$

$$\Rightarrow z = \frac{1}{7}\left[\pm\frac{13}{2}\left(\frac{7\sqrt{13} - 13}{117}\right) - 2\right]$$

Thus, there are two possible choices of each x, y, z

$\Rightarrow$ There are 6 total critical points.

Plug in each of the points into f to find points which give max and min distance.