# Math 241 Homework 1 Solution

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Problems are 5 pts each.

1. Problem 2.14.4 (2pts 1pt each)
   1) c, d, e
   2) b, c, d, e
   Problem 2.14.7 (3pts 1pt each)
   $v_1 = \begin{bmatrix}2 & 3 & -4 & 1\end{bmatrix}$, $v_2 = \begin{bmatrix}1 & -5 & 2 & 0\end{bmatrix}$, $v_3 = \begin{bmatrix}4 & 1 & -1 & -1\end{bmatrix}$

2. Problem 3.8.4 (2pts 1pt each)
   $v_1 = \begin{bmatrix}1 & 0 & a\end{bmatrix}$, $v_2 = \begin{bmatrix}0 & 1 & b\end{bmatrix}$
   (Any scalar multiples of $v_1$ and $v_2$ is also correct.)
   Claim: for any set of points $v = [x, y, z]$ satisfying the equation $z = ax+by$, $v$ can be written as a linear combination of $v_1$ and $v_2$, aka $\exists c, d \in \mathbb{R}$ s.t. $v = cv_1 + dv_2$.
   Proof: (-0.5pt minor error / -1pt major error)
   Take $c = x \in \mathbb{R}$ and $d = y \in \mathbb{R}$, $cv_1 = [x, 0, ax]$ and $dv_2 = [0, y, by]$, thus $cv_1 + dv_2 = [x + 0, 0 + y, ax + by] = [x, y, ax + by] = [x, y, z] = v$.
   Problem 3.8.5 (3pts 1pt each)
   $v_1 = \begin{bmatrix}1 & 0 & a\end{bmatrix}$, $v_2 = \begin{bmatrix}0 & 1 & b\end{bmatrix}$, $v_3 = \begin{bmatrix}0 & 0 & c\end{bmatrix}$
   Proof is similar to Problem 3.8.4. (Point allocation same to previous)

3. Free 1 pt, 4pts 1pt each
   Use Back Substitution because the system of linear equations is in upper triangular form and $a_{kk} \neq 0$ , $\forall k = 1, \cdots, 4$.
   We get $x_1 = 3,\ x_2 = -1,\ x_3 = \frac{1}{2},\ x_4 = -2$.

4. 0.03125 2pts 0.291667 1.5pts 0.40 1.5pts
   $\mathbf{w}=\begin{bmatrix}0.03125 & \cdots & 0.03125 & 0.35 \times \frac{100}{120} = \frac{7}{24} = 0.2916666 & 0.40 \times \frac{100}{160} = \frac{1}{4} = 0.25\end{bmatrix}$.

5. expanding 2pts
   u + v = v + u 2pts
   citing commutativity of real numebrs 1pt
   Claim: $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ where $u, v \in \mathbb{R}^n$
   Proof: Let $u = \begin{bmatrix}u_1 \\ u_2 \\ \vdots \\ u_n\end{bmatrix}$, $v = \begin{bmatrix}v_1 \\ v_2 \\ \vdots \\ v_n\end{bmatrix}$.
   Then $u+v = \begin{bmatrix}u_1 \\ u_2 \\ \vdots \\ u_n\end{bmatrix} + \begin{bmatrix}v_1 \\ v_2 \\ \vdots \\ v_n\end{bmatrix} = \begin{bmatrix}u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n\end{bmatrix}$ by defn of vector addition
   $= \begin{bmatrix}v_1 + u_1 \\ v_2 + u_2 \\ \vdots \\ v_n + u_n\end{bmatrix}$ by commutativity of real numbers.
   $= v + u$ by defn of vector addition.

6. m=2 case 4pts
   larger m 1pt
   For the case that $m = 2$. Assume $b_1 = da_1 + ea_2$, $b_2 = fa_1 + ga_2$, and $c = hb_1 + ib_2$ where $d, e, f, g, h, i \in \mathbb{R}$. Then, $c = hb_1 + ib_2 = h * (da_1 + ea_2) + i * (fa_1 + ga_2) = (h * d + i * f)a_1 + (h * e + i * g)a_2$. Since $\mathbb{R}$ is closed under addition and multiplication, $h * d + i * f, h * e + i * g \in \mathbb{R}$, thus $c$ is a linear combination of $a_1$ and $a_2$.
   Proof for larger $m$ is similar to above.