Math 241 Homework

Mary Radcliffe

Due 6 September 2018

Complete the following problems. Fully justify each response.

1. Complete problems 2.14.4, 2.14.7 on page 111 of Coding the Matrix. (Note: Klein uses bitstrings to represent vectors over $\mathbb{Z}_2$. So for example, 0011 would denote the vector $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.)

2. Complete problems 3.8.4, 3.8.5 on page 146 of Coding the Matrix.

3. Use ideas discussed in class to solve the following upper triangular system of linear equations:

$$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & - & 2x_3 & + & 2x_4 & = & -5 \\ & & 3x_2 & + & 4x_3 & - & x_4 & = & 1 \\ & & & & 2x_3 & + & x_4 & = & -1 \\ & & & & & & -5x_4 & = & 10 \end{array}$$

4. (ALA Problem 1.10) The record for each student in a class is given as a 10-vector $\mathbf{r}$, where $r_1, r_2, \ldots, r_8$ are the scores for the 8 homework assignments, each on a 0-10 scale, r_9 is the midterm exam grade on a 0-120 scale, and r_{10} is the final exam score on a 0-160 scale. The student's total course score s, on a 0-100 scale, is based on a weighted average of 25% homework, 35% midterm, and 40% final exam. Determine a vector $\mathbf{w}$ so that $s = \mathbf{w} \cdot \mathbf{r}$.

5. In lecture, we proved that vector addition is associative. Prove that vector addition is also commutative; that is, prove that $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$. For the purposes of this problem, you may assume that your vectors are over $\mathbb{R}$, and you may use standard arithmetic rules in $\mathbb{R}$.

6. (Similar to ALA Problem 1.18) Suppose that each of the vectors $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_k$ is a linear combination of the vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_m$, and that $\mathbf{c}$ is a linear combination of $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_k$. Then $\mathbf{c}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_m$. Show this for the case that $m = 2$. Explain why it would also be true if m is larger. (The proof is not hard, but the notation is a mess)

7. Read through Coding the Matrix 2.9.4-2.9.7. Here, you will find a discussion of dot products over $\mathbb{Z}_2$, and how we can use $\mathbb{Z}_2$ for authentication schema.

8. Complete the first problem set found at `autolab.andrew.cmu.edu`. The submission for this is directly on autolab, no need to hand it in on paper.