21-241 Assignment 2 (20Pt total)

1:

(3.8.8)

1. show that $\vec{0}$ is in W. (**1 pt**)
   $[x, y, z] = [0, 0, 0] \Rightarrow x + y + z = 0 + 0 + 0 = 0$. Therefore $\vec{0} \in W$.

2. show that addition holds. (**1 pt**)
   Suppose that $[x_1, y_1, z_1], [x_2, y_2, z_2] \in W$, then $x_1 + y_1 + z_1 = 0$ and $x_2 + y_2 + z_2 = 0$.
   $[x_1, y_1, z_1] + [x_2, y_2, z_2] = [x_1 + x_2, y_1 + y_2, z_1 + z_2]$.
   $x_1 + x_2 + y_1 + y_2 + z_1 + z_2 = x_1 + y_1 + z_1 + x_2 + y_2 + z_2 = 0 + 0 = 0$.
   so $[x_1, y_1, z_1] + [x_2, y_2, z_2] \in W$

3. show that scalar multiplication holds. (**1 pt**)
   Suppose that $[x, y, z] \in W$, then $x + y + z = 0$.
   $c[x, y, z] = [cx, cy, cz]$.
   $cx + cy + cz = c(x + y + z) = c * 0 = 0$.
   so $c[x, y, z] \in W$

(3.8.9) Counterexample on addition. (**Each vector 1 pt**). As long as it make sense.
E.g.
$[0, 1, 0, 0, 0]$ and $[0, 0, 0, 0, 1]$ are both in W.
$[0, 1, 0, 0, 0] + [0, 0, 0, 0, 1] = [0, 1, 0, 0, 1]$ is not in W.

2:

(4.17.2)(**Matrix 1 pt, matrix multiplication 1pt**)

$$M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$M * [x, y] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * [x, y] = [0x + 1y, 1x + 0y] = [y, x]$$

(4.17.3)(**Matrix 2 pt, matrix multiplication 1pt**)

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M * [x, y, z] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} * [x, y, z] = [1x + 0y + 1z, 0x + 1y + 0z, 1x + 0y + 0z] = [x + z, y, x]$$

3:

(a) (**correct matrix multiplication 1pt, description in word 1pt**)
$\vec{v} = [v_1, v_2, v_3]$

$$Av = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} * [v_1, v_2, v_3] = [0 + 0 + v_3, 0 + v_2 + 0, 0 + 0 + v_1] = [v_3, v_2, v_1]$$

Flip the vector v.

(b) (**correct matrix multiplication 1pt**)
$\vec{v} = [v_1, v_2, v_3]$

$$Av = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix} * [v_1, v_2, v_3, v_4, v_5, v_6]$$

$$Av = [1/2(v_1 + v_2), 1/2(v_1 + v_2), 1/2(v_3 + v_4), 1/2(v_3 + v_4), 1/2(v_5 + v_6)]$$

(c) (**correct matrix multiplication 1pt, description in word 1pt**)
$\vec{v} = [v_1, v_2, v_3]$

$$Av = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * [v_1, v_2, v_3] = [v_1 + v_2 + v_3, v_1 + v_2 + v_3, v_1 + v_2 + v_3]$$

Put the sum of entries in each position.

4:

Expansion of M(u+v): 2pt, Expansion of Mu, Mv: 2pt, Mu+Mv: 1pt

Let $M = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{d2} & x_{d3} & \dots & x_{mn} \end{bmatrix}$

$u = [u_1, u_2, \vdots, u_n]$, $v = [v_1, v_2, \vdots, v_n]$

Then $u + v = [u_1 + v_1, u_2 + v_2, \vdots, u_n + v_n]$

$$M(u+v) = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{d2} & x_{d3} & \dots & x_{mn} \end{bmatrix} * [u_1 + v_1, u_2 + v_2, \vdots, u_n + v_n]$$
$$= \begin{bmatrix} x_{11}(u_1+v_1) + x_{12}(u_2+v_2) + \cdots + x_{1n}(u_n+v_n) \\ x_{21}(u_1+v_1) + x_{22}(u_2+v_2) + \cdots + x_{2n}(u_n+v_n) \\ \vdots \\ x_{m1}(u_1+v_1) + x_{m2}(u_2+v_2) + \cdots + x_{mn}(u_n+v_n) \end{bmatrix}$$

$$Mu + Mv = \begin{bmatrix} x_{11}u_1 + x_{12}u_2 + \cdots + x_{1n}u_n \\ x_{21}u_1 + x_{22}u_2 + \cdots + x_{2n}u_n \\ \vdots \\ x_{m1}u_1 + x_{m2}u_2 + \cdots + x_{mn}u_n \end{bmatrix} + \begin{bmatrix} x_{11}v_1 + x_{12}v_2 + \cdots + x_{1n}v_n \\ x_{21}v_1 + x_{22}v_2 + \cdots + x_{2n}v_n \\ \vdots \\ x_{m1}v_1 + x_{m2}v_2 + \cdots + x_{mn}v_n \end{bmatrix}$$
$$= \begin{bmatrix} x_{11}(u_1+v_1) + x_{12}(u_2+v_2) + \cdots + x_{1n}(u_n+v_n) \\ x_{21}(u_1+v_1) + x_{22}(u_2+v_2) + \cdots + x_{2n}(u_n+v_n) \\ \vdots \\ x_{m1}(u_1+v_1) + x_{m2}(u_2+v_2) + \cdots + x_{mn}(u_n+v_n) \end{bmatrix}$$
$$= M(u+v)$$