Math 241 Homework

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Due 4 October 2018

Complete the following problems. Fully justify each response.

1. Complete problems 4.17.8-9 on page 208-209 of Coding the Matrix.

2. Find the standard matrix for each of the following linear transformations from $\mathbb{R}^2 \to \mathbb{R}^2$. Determine if each transformation is injective, surjective, both or neither. Explain your response.

   (a) Reflection over the line $y = x$

   (b) Reflection over the line $y = -x$

   (c) Projection onto the line $y = x$

3. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that the jth row of AB is the product of the jth row of A (viewed as a $1 \times n$ matrix) with B.

4. Determine if the following statement is true or false:

   > Let A, X, Y be $n \times n$ matrices, satisfying $AX = AY$. Then $X = Y$.

   If the statement is true, prove it. If the statement is false, give an example showing why, and add a hypothesis that would make it true.

5. Suppose A is an invertible matrix. Prove that A^T is also invertible, and $(A^T)^{-1} = (A^{-1})^T$ (this is sometimes denoted by A^{-T}).

6. Prove that a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if $ad - bc \neq 0$. (Hint: you can do this by demonstrating an inverse).

7. Complete the fourth problem set found at `autolab.andrew.cmu.edu`. The submission for this is directly on autolab, no need to hand it in on paper.