Math 241 Homework

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Due 6 November 2018

Complete the following problems. Fully justify each response.

This homework set can be turned in during lecture on 5 November, or may be turned in to Mary's office between 11:00 and 2:00 on Tuesday, 6 November. If these turn in times are not possible for you, please speak to your TA about an alternative.

1. Define a projection $P : \mathbb{R}^n \to \mathbb{R}^n$ to be a linear transformation such that $P^2 = P$.
   (a) Prove that an orthogonal projection onto a subspace W is a projection, as defined above.
   (b) Give an example of a projection that is NOT an orthogonal projection onto a subspace.
   (c) What are the possible eigenvalues of a projection? Explain.
2. Complete problem 8.6.3 on page 350 of Coding the Matrix.
3. Prove that a set of mutually orthogonal, nonzero vectors is linearly independent.
4. Let W be a subspace of $\mathbb{R}^n$. Define $W^\perp = \{\mathbf{v} \in \mathbb{R}^n \mid \mathbf{v} \perp W\}$. Prove that if $\mathcal{B}_W$ is a basis for W, and $\mathcal{B}_{W^\perp}$ is a basis for $W^\perp$, then $\mathcal{B}_W \cup \mathcal{B}_{W^\perp}$ is a basis for $\mathbb{R}^n$. (Hint: I would NOT use a dimension argument here. Instead, try an argument about unique representation of every vector in $\mathbb{R}^n$)
5. Let λ_1, λ_2 be two different eigenvalues for a symmetric matrix A, and let $\mathcal{E}(\lambda_1), \mathcal{E}(\lambda_2)$ be the corresponding eigenspaces. Prove that if $v \in \mathcal{E}(\lambda_2)$, then $v \perp \mathcal{E}(\lambda_1)$. Use this to show that if A has a set of n linearly independent eigenvectors, it also has a set of n orthogonal eigenvectors.
6. (Optional turnin) Compute the following projections.
   (a) projection of $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ onto Span $\left\{ \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} \right\}$
   (b) projection of $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ onto Span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right\}$
7. (Optional turnin) Find an orthogonal basis for each of the following.
   (a) The plane in $\mathbb{R}^3$ through the origin whose equation can be written as $2x + 3y - z = 0$.

(b) The hyperplane (a 3-dimensional space) in $\mathbb{R}^4$ through the origin whose equation can be written as $x_1 + 2x_2 - x_4 = 0$.

8. Complete the problem set found at `autolab.andrew.cmu.edu`. The submission for this is directly on autolab, no need to hand it in on paper.