Math 241 Homework

Mary Radcliffe

Due 6 December 2018

Complete the following problems. Fully justify each response.

1. Complete problems 12.2 and 12.3 on page 239 in Introduction to Applied Linear Algebra (ALA) by Stephen Boyd and Lieven Vandenberghe. (Note: when we think about angles between vectors, you may use the property, not proved in this class, that if θ is the angle between $\mathbf{x}$ and $\mathbf{y}$, then $\mathbf{x}^T\mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\|\cos\theta$.)

2. Complete problems 13.7, 13.8, 13.10 on pages 280-281 in Introduction to Applied Linear Algebra (ALA) by Stephen Boyd and Lieven Vandenberghe.

3. Let A be an $m \times n$ matrix with singular value decomposition $A = U\Sigma V^T$, where U is $m \times m$, Σ is $m \times n$, and V is $n \times n$. Write the columns of U as $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m$, the columns of V as $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$, and let $k = \text{rank}\,(A)$. Let $\mathbf{b} \in \mathbb{R}^m$, and write $\mathbf{b} = \alpha_1\mathbf{u}_1 + \cdots + \alpha_m\mathbf{u}_m$ as a linear combination of the columns of U (which are a basis for $\mathbb{R}^m$.

   Prove that the least squares solution $\hat{\mathbf{x}}$ to the equation $A\mathbf{x} = \mathbf{b}$ satisfies $A\hat{\mathbf{x}} = \alpha_1\mathbf{u}_1 + \cdots + \alpha_k\mathbf{u}_k$; that is, it is the terms in $\mathbf{b}$ corresponding to the nonzero singular values of A. Show that $\hat{\mathbf{x}} = \frac{\alpha_1}{\sigma_1}\mathbf{v}_1 + \cdots + \frac{\alpha_k}{\sigma_k}\mathbf{v}_k$ satisfies this equation.

   (Notice that in the above analysis, we did NOT rely on independence or dependence of the columns of matrix A. So we can use the SVD for A to give solutions to a least squares problem regardless of these properties).

4. Complete the problem set found at `autolab.andrew.cmu.edu`. The submission for this is directly on autolab, no need to hand it in on paper.