Math 127 Homework

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Complete the following problems. Fully justify each response. You need only turn in those problems marked with a (*).

1. (*) Prove De Morgan's Laws as stated in Theorem 2 in the Propositional Logic notes.
2. Prove that $p \Leftrightarrow q$ is logically equivalent to $(p \Rightarrow q) \wedge (q \Rightarrow p)$.
3. (*) Write the following statement as a proposition using propositional variables and logical operators. You may assume that "x is odd" is the logical negation of "x is even."

   Let x be a positive integer. Then $x = 2$, or x is even and $x > 2$, or x is odd and prime, or x is odd and composite.

4. (*) Let p, q, r be propositions. Show that
   $$\neg\left[(p \wedge q) \vee (p \wedge (\neg r)) \vee (q \wedge r)\right]$$
   is logically equivalent to
   $$\neg(q \wedge r) \wedge ((\neg p) \vee r).$$
5. Show that $(p \Rightarrow q) \vee (\neg q)$ is a tautology.
6. Let x, y be rational numbers. Use the method of direct proof to show that xy, x/y, and $x - y$ are all rational numbers.
7. (*) Prove the following proposition:

   Let d, a, b, u, v be positive integers. If a is divisible by d and b is divisible by d, then $au + bv$ is divisible by d.

8. (*) Follow the strategy of Example 4 in the Logic and Proof notes to prove the Quadratic formula:

   A complex number x is a solution to the equation $ax^2 + bx + c = 0$ if and only if
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$