Name: ______________________________

Recitation (circle one): F(10:30) G(12:30) H(1:30) J(3:30)

Math 127: Exam 1
14 February 2018

Turn off and put away your cell phone.
No notes or books are permitted during this exam.
No calculators or any other devices are permitted during this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write solutions clearly and legibly; credit will not be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Σ	100	

1. Consider the following statement:

$$\forall x \in \mathbb{R}, \exists ! y \in \mathbb{R}, xy = 0$$

   (a) Write this statement as a sentence, without using any of the symbols $\forall, \exists, !, \in$ or $\mathbb{R}$.

   (b) Is this statement true? If so, prove it. If not, explain why not.

   (c) If we reversed the order of the quanitifiers, would the statement be true? If so, prove it. If not, explain why not.

2. (a) Each of the propositions on the right is logically equivalent to one on the left. Match them by writing the appropriate number in the blank. You need not justify your answers in any way.

- p ____ i. $(p \wedge q) \vee \neg(p \vee q)$
- $p \Leftrightarrow q$ ____ ii. $\neg((\neg p) \wedge (\neg q))$
- $p \Rightarrow q$ ____ iii. $(\neg p) \vee q$
- $p \vee q$ ____ iv. $p \wedge ((\neg p) \vee q)$
- $p \wedge q$ ____ v. $p \vee (p \wedge q)$

(b) Prove that $(p \vee q) \Rightarrow r$ is logically equivalent to $(p \Rightarrow r) \wedge (q \Rightarrow r)$.

3. Find all real-valued solutions to the equation

$$\sqrt{x^2 - 3x} = \sqrt{2 - 2x}.$$

Prove that your answer is correct.

4. Let $k \in \mathbb{N}$, with $k \geq 1$. Prove that for all $n \in \mathbb{N}$ with $n \geq 1$,

$$\sum_{i=1}^{n} \binom{i+k-1}{k} = \binom{n+k}{k+1}.$$

5. Define a sequence (a_k) recursively by

$$a_0 = 1,$$

$$a_1 = 1,$$

$$a_k = a_{k-1} + a_{k-2} \text{ for } k \geq 2.$$

Prove that for all $k \geq 1$, $a_k \geq \left(\frac{3}{2}\right)^{k-2}$.