21-127 Final Exam Study Guide

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Here are major topics for your final exam! Please let me know if you have any questions.

1 Basic proof techniques

1. Straightforward conditional: proving statements of the form $P \Rightarrow Q$ by assuming P and deriving Q.
2. Biconditional: proving statements of the form $P \Leftrightarrow Q$.
3. Proof by contradiction: proving statements of the form $P \Rightarrow Q$ by assuming P and $\neg Q$ and deriving a contradiction.
4. Proof by cases: proving statements of the form $P \vee Q \Rightarrow R$ by taking two cases: $P \Rightarrow R$ and $Q \Rightarrow R$.
5. Proof by contrapositive: proving statements of the form $P \Rightarrow Q$ by assuming $\neg Q$ and deriving $\neg P$.
6. Law of Excluded Middle
7. Induction: weak and strong

2 Propositional Logic

1. Understanding propositional formulae: propositional variables, conjunction, disjunction, negation, conditional
2. De Morgan's Laws
3. Quantifiers, ordering of quantifiers, negations

3 Set Theory and functions

1. Basic definitions, set builder notation
2. Unions, intersections, complements, power sets, Cartesian products
3. Proving equality between sets using double containment
4. De Morgan's Laws
5. Basic definitions for functions: domain, codomain, graph, injective, surjective, bijective, image, preimage, compositions, left/right/two-sided inverses
6. Well-definedness: definitions, how to prove

4 Divisibility and Number Theory

1. Division Theorem
2. GCDs: basic definitions and theorems
3. Euclidean Algorithm
4. Bezout's Lemma
5. Primes: basic definitions
6. Fundamental Theorem of Arithmetic
7. Base n expansion

5 Modular arithmetic

1. Basic definitions of equivalence
2. Arithmetic: how to perform basic operations
3. Multiplicative inverses: when they exist, and how to find them
4. Fermat's Little Theorem, Euler's Totient Theorem
5. Wilson's Theorem
6. Understanding of modular equivalence as an equivalence relation, and $\mathbb{Z}_n$; re-understanding of the equivalence classes as a field when n is prime

6 Counting: finite

1. Definition of finiteness
2. Basic counting theorems: Products, unions, intersections
3. Inclusion-Exclusion
4. Permutations and binomial coefficients, Binomial Theorem
5. Counting in 2 ways
6. Counting by bijection

7 Counting: infinite

1. Definition of two sets having the same size
2. Definition of countably infinite, uncountably infinite
3. Proof that the finite product of countable sets is countable
4. Proof that the countable union of countable sets is countable
5. Proof that $\mathbb{Q}$ is countable
6. Cantor's Diagonalization

8 Equivalence Relations

1. Basic definitions of relations, equivalence relations
2. Understanding of equivalence relations via partition of ground set
3. Understanding of equivalence relations via functions

9 Posets

1. Basic definitions of poset as a relation
2. Key terminology: minimum, minimal, maximum, maximal, supremum (aka LUB, aka join), infimum (aka GLB, aka meet), lattice, distributive lattice, complement
3. Understanding of Hasse diagram for representing poset structure

10 Fundamentals of sequences and convergence

1. Understanding of what a field is (axioms will be given), and what an ordered field is
2. Distances, Triangle Inequality, Cauchy-Schwarz Inequality
3. Definition of convergence/divergence of a real sequence, and ability to use/manipulate that definition to prove if a sequence converges or diverges.
4. Monotone Convergence Theorem
5. Squeeze Theorem

11 Fundamentals of discrete probability

1. Definition of a discrete probability space, and basic manipulation of probabilities
2. Basics of conditional probability and independence: law of total probability, Bayes' Theorem
3. Definitions of random variables
4. Basics of distribution: definitions, pmfs for important distributions
5. Expectation: what it is, how to calculate